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WIGNER'S INFLUENCE ON PARTICLE PHYSICS: UNIFICATION OF SPACETIME SYMMETRIES OF MASSIVE AND MASSLESS PARTICLES*

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The internal space–time symmetries of relativistic particles are dictated by Wigner's little groups. The $O(3)$-like little group for a massive particle at rest and the $E(2)$-like little group of a massless particle are two different manifestations of the same covariant little group. Likewise, the quark model and parton pictures are two different manifestations of the one covariant entity.

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1. Introduction

Eugene Wigner's 1939 paper on the Poincaré group is regarded as one of the most fundamental papers in modern physics [1]. Wigner observed there that relativistic particles have their internal space-time degrees of freedom, and formulated their symmetries in terms of the little groups of the Poincaré group. He then showed that the little groups for massive and massless particles are isomorphic to the $O(3)$ and $E(2)$ groups respectively.

The purpose of this report is to emphasize that the little group is a Lorentz-covariant entity and unifies the internal space-time symmetries of both massive and massless particles, just as Einstein's $E = mc^2$ does for the energy-momentum relation. On the other hand, Wigner did not reach this conclusion in 1939, but his paper raised the following questions.

1 Like the three-dimensional rotation group, $E(2)$ is a three-parameter group. It contains two translational degrees of freedom in addition

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to the rotation. What physics is associated with the translational-like degrees of freedom for the case of the $E(2)$-like little group?

2 As is shown by Inonu and Wigner [2], the rotation group $O(3)$ can be contracted to $E(2)$. Does this mean that the $O(3)$-like little group can become the $E(2)$-like little group in a certain limit?

3 It is possible to interpret the Dirac equation in terms of Wigner's representation theory [3]. Then, why is it not possible to find a place for Maxwell's equations in the same theory?

4 The proton had been known to have a finite space-time extension, and it is believed to be a bound state of quarks. Is it then possible to construct a representation of the Poincaré group for particles with space-time extensions?

As for the first question, it has been shown by various authors that the translation-like degrees of freedom in the $E(2)$-like little group is the gauge degree of freedom for massless particles[4, 5]. The second question will be addressed in detail in Sec. 2. As for the third question, Weinberg found a place for the gauge-invariant electromagnetic fields in the Wigner formalism by constructing from the SL(2,c) spinors all the representations of massless fields which are invariant under gauge transformations [6]. It has also been shown that gauge-dependent four-potentials can also be constructed within the SL(2,c) framework [7]. The Maxwell theory and the Poincaré group are now perfectly consistent with each other.

The fourth question is about whether Wigner's little groups are applicable to high-energy particle physics where accelerators produce Lorentz-boosted extended hadrons such as high-energy protons. The question is whether it is possible to construct a representation of the Poincaré group for hadrons which are believed to be bound states of quarks [5, 8]. This representation should describe Lorentz-boosted hadrons. Next question then is whether those boosted hadrons give a description of Feynman's parton picture [9] in the limit of large momentum/mass [5, 10]. We shall concentrate on this fourth question in this report.

2. Little Groups of the Poincaré Group

The little group is the maximal subgroup of the Lorentz group which leaves the four-momentum invariant. While leaving the four-momentum invariant, the little group governs the internal space-time symmetries of relativistic particles. The Lorentz group is generated by three rotation generators J_i and three boost generators K_i. If a massive particle is at rest, the little group is the three-dimensional rotation group generated by J_1, J_2 and J_3. The four-momentum is not affected by this rotation, but the spin

variable changes its direction. For a massless particle moving along the z direction, Wigner observed that the little group is generated by J_3, N_1 and N_2, where

$$N_1 = J_1 + K_2, \qquad N_2 = J_2 - K_1\,, \tag{1}$$

and that these generators satisfy the Lie algebra for the two-dimensional Euclidean group. Here, J_3 is like the rotation generator, while N_1 and N_2 are like translation generators in the two-dimensional Euclidean plane.

In 1953, Inonu and Wigner formulated this problem as the contraction of $O(3)$ to $E(2)$. How about then the little groups which are isomorphic to $O(3)$ and $E(2)$? It is reasonable to expect that the $E(2)$-like little group be obtained as a limiting case for of the $O(3)$-like little group for massless particles [11]. It is shown that, under the boost along the z direction, the rotation generator around the z axis remains invariant, but the transverse rotation generators J_1 and J_2 become generators of gauge transformations in the large-boost limit [12]. It was later shown that the little group for massless particles has the geometry of the cylindrical group [13].

In the following sections, we shall discuss how the concept of little groups are applicable to space-time symmetries of relativistic extended hadrons.

3. Covariant Harmonic Oscillators

Let us consider a hadron consisting of two quarks. If the space-time position of two quarks are specified by x_a and x_b respectively, the system can be described by the variables

$$X = \frac{(x_a + x_b)}{2}\,, \qquad x = \frac{(x_a - x_b)}{2\sqrt{2}}\,. \tag{2}$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks.

The portion of the wave function which is subject to the $O(3)$-like little group takes the form

$$\psi_0^n(z,t) = \left(\frac{1}{\pi n! 2^n}\right)^{1/2} H_n(z) \exp\left\{-\frac{1}{2}\left(z^2 + t^2\right)\right\}\,. \tag{3}$$

The subscript 0 means that the wave function is for the hadron at rest. The above expression is not Lorentz-invariant, and its localization undergoes a Lorentz squeeze as the hadron moves along the z direction [5].

It is convenient to use the light-cone variables to describe Lorentz boosts. The light-cone coordinate variables are

$$u = \frac{(z+t)}{\sqrt{2}}\,, \qquad v = \frac{(z-t)}{\sqrt{2}}\,. \tag{4}$$

In terms of these variables, the Lorentz boost takes the simple form

$$u' = e^{\eta} u\,, \qquad v' = e^{-\eta} v\,, \tag{5}$$

where η is the boost parameter and is $\tanh^{-1}(v/c)$. This is a "squeeze" transformation.

In Eq. (3), the localization property of the wave function is determined by the Gaussian factor, and we shall therefore study the ground state and its wave function

$$\psi_0(z,t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(u^2+v^2)\right\}. \tag{6}$$

If the system is boosted, the wave function becomes

$$\psi_\eta(z,t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left(e^{-2\eta}u^2 + e^{2\eta}v^2\right)\right\}. \tag{7}$$

The wave function of Eq. (6) is distributed within a circular region in the uv plane, and thus in the zt plane. On the other hand, the wave function of Eq. (7) is distributed in an elliptic region whose major and minor axes are along the light-cone axes. The wave function becomes Lorentz-squeezed!

4. Feynman's Parton Picture

In order to explain the scaling behavior in inelastic scattering, Feynman in 1969 observed that a fast-moving hadron can be regarded as a collection of many "partons" whose properties do not appear to be identical to those of quarks [9]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman made the following systematic observations.

a The picture is valid only for hadrons moving with velocity close to that of light.

b The interaction time between the quarks becomes dilated, and partons behave as free independent particles.

c The momentum distribution of partons becomes widespread as the hadron moves fast.

d The number of partons seems to be infinite or much larger than that of quarks.

Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly b) and c) together. In order to resolve this paradox, we need a momentum–energy wave function. If the quarks have the four-momenta p_a and p_b, we can construct two independent four-momentum variables [8]

$$P = p_a + p_b\,, \qquad q = \sqrt{2}(p_a - p_b)\,. \tag{8}$$

The four-momentum P is the total four-momentum and is thus the hadronic four-momentum, while q measures the four-momentum separation between the quarks. In the light-cone coordinate system, the momentum-energy variables are

$$q_u = \frac{(q_0 - q_z)}{\sqrt{2}}\,, \qquad q_v = \frac{(q_0 + q_z)}{\sqrt{2}}\,. \tag{9}$$

Then the momentum-energy wave function takes the form

$$\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\tfrac{1}{2}\left(e^{-2\eta}q_u^2 + e^{2\eta}q_v^2\right)\right\}\,. \tag{10}$$

The momentum wave function is also squeezed, and the parton momentum distribution becomes wide-spread as the hadronic speed approaches the speed of light. It is thus possible to calculate the parton distribution by boosting a hadronic wave function in the rest frame. The calculation based on this oscillator model gives a reasonable agreement with the measured parton distribution function [14].

Let us go back to the Lorentz-squeezed space-time wave function given in Eq. (7). This wave function gives two time intervals corresponding to the major and minor axes of the elliptic distribution. As the hadronic speed approaches the speed of light, the major axis corresponds to the period of oscillation, and it increases by factor of e^η. This period measures the interaction time among the quarks.

The external signal comes into the hadron in the direction opposite to the hadron momentum. Thus the minor axis of the ellipse measures the the time the external signal spends inside the hadron. This is the interaction time between one of the quark and the external signal. This time interval decreases as $e^{-\eta}$. The ratio of the interaction time to the oscillator period becomes $e^{-2\eta}$. The energy of each proton coming out of the Fermilab accelerator is 900 GeV. This leads the ratio to 10^{-6}, which is indeed a small number. The external signal is not able to sense the interaction of the quarks among themselves inside the hadron. This is why partons appear as free particles with a wide-spread momentum distribution.

The internal space-time symmetry of hadrons in the quark model can be framed into the $O(3)$-like little group when they are slowly moving particles.

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