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Analyticity Requirement for Regge Poles and Backward Unequal-Mass Scattering II

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ABSTRACT

We evaluate exactly the modified Regge amplitude for the backward unequal-mass scattering. This gives the Regge behavior $u^{\alpha(s)}$ at $s=0$ and in its neighborhood. We then consider a possibility that a fixed pole contribution is completely eliminated by a counteracting Regge amplitude. It is shown that, if such an amplitude is added, the total Regge amplitude vanishes at $s=0$.

In a previous paper,[1] a Regge amplitude satisfying the Mandelstam representation is constructed for the backward unequal-mass scattering. Based on the two lowest-order coefficients in a power series expansion in s, it was shown that the leading term gives the Regge behavior $u^{\alpha(s)}$ at $s=0$ and in its neighborohood including the region in which the cosine of the backward scattering angle is bounded. It was shown also that the coefficients of the non-leading terms do not give rise to a singularity of the Regge amplitude.

In this paper, we obtain the exact s dependence of the asymptotic form of the Regge amplitude for large u, that is, we compute the coefficients of the $u^{\alpha(s)}$, $u^{\alpha(s)-1}$ and $u^{\alpha(0)-1}$ terms exactly. We confirm the assertions made in the previous paper.[1] We then consider a possibility of eliminating the fixed power term $u^{\alpha(0)-1}$ by introducing another Regge pole with $u^{\bar{\alpha}(s)}$, $u^{\bar{\alpha}(s)-1}$ and $u^{\bar{\alpha}(0)-1}$ terms. It is shown that this elimination procedure leads to a vanishing total Regge amplitude at $s=0$.

In the previous paper,[1] it was shown that the analyticity approach leads to the following expression for the modified. Regge amplitude.

$$K(s,u)=\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ImR(s',u)\,ds'}{s'-s}, \tag{1}$$

where

$$R(s',u)=-\pi\gamma(s')\,(-q'^2)^{\alpha(s')}P_{\alpha(s')}\times\left(-1-\frac{u-r^2/s'}{2q'^2}\right),$$

$$q'^2=\frac{[s'-(m-\mu)^2][s'-(m+\mu)^2]}{4s'},$$

and

$$s_0=(m+\mu)^2$$

m and μ are the nucleon and pion masses respectively. As in all previous papers on this subject we are dealing here with pion-nucleon backward scattering.

The Regge amplitude $R(s',u)$, in addition to the physical cut, has a cut running from $s'=0$ to r^2/u in the complex s' plane, Therefore the above dispersion integral can be replaced by a contour integral which encloses the pole of the integrand at $s'=s$ and a cut running from 0 to r^2/u counterclockwise.

$$K(s,u)=\frac{1}{2\pi i}\int_C ds'\frac{R(s',u)}{s'-s}. \tag{2}$$

C is the contour described above. We can choose this contour in such a way that the following conditions are satisfied:

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(1) $\alpha(s')$ and $\gamma(s')$ are analytic within the contour.

(2) $q'^2 = r^2/4s'$ along the contour.

(3) $$P_{\alpha(s')}\left(1-\frac{2us'}{r^2}\right)=\frac{1}{\sqrt{\pi}}\frac{\Gamma[\alpha(s')+]1/2}{\Gamma[\alpha(s')+1]}\times\left[u^{\alpha(s)}-\frac{\alpha(s')r^2}{2s'}u^{\alpha(s')-1}\right] \quad (3)$$

along the contour.

Then

$$R(s',u)=-\sqrt{\pi}\,\gamma(s')\frac{\Gamma[\alpha(s')+1/2]}{\Gamma[\alpha(s')+1]}\times\left[u^{\alpha(s')}-\frac{\alpha(s')r^2}{2s'}u^{\alpha(s')-1}\right]. \quad (4)$$

According to the above expression for $R(s', u)$, the integrand of Eq. (2) has poles at $s'=s$ and at $s'=0$. Now, by taking residues we obtain

$$K(s,u) = -\sqrt{\pi}\,\gamma(s)\frac{\Gamma[\alpha(s)+1/2]}{\Gamma[\alpha(s)+1]}\left[u^{\alpha(s)}-\frac{\alpha(s)r^2}{2s}u^{\alpha(s)-1}\right] -\sqrt{\pi}\,\gamma(0)\frac{\Gamma[\alpha(0)+1/2]}{\Gamma[\alpha(0)+1]}\left[\frac{\alpha(0)r^2}{2s}u^{\alpha(0)-1}\right]. \quad (5)$$

The leading term in Eq. (5) is $u^{\alpha(s)}$ as has been expected. The above $K(s, u)$ also gives a fixed-pole contribution $\frac{1}{s}u^{\alpha(0)-1}$. This term has a pole at $s=0$. But this singularity is canceled by the $\frac{1}{s}u^{\alpha(s)-1}$ term which also exists in the right hand side of Eq. (5). Unfortunately, Freedman *et al*[2]. did not realize that these poles canecl each other at $s=0$ and asserted the existence of a daughter trajectory which will remove the singularity of the fixed power term. We may mention further that $K(s, u)$ of Eq. (5) reduces to that of the previous paper[1] in the small s limit.

It is by now very clear that the daughter trajectory of Freedman *et. ai.* type[2] does not exist. But, let us consider a possibility of eliminating completely the fixed-pole contribution by introducing a counteracting Regge pole, that is, we construct a new amplitude $\bar{K}(s, u)$ with the parameters $\bar{\alpha}(s)$ and $\bar{\gamma}(s)$, add to the original amplitude:

$$K_{tot}(s,u) \equiv K(s,u)+\bar{K}(s,u),$$

and insist that $K_{tot}(s, u)$ contains no fixed pole terms. Then from Eq. (5) it is clear that $\alpha(0)=\bar{\alpha}(0)$. In other words, the two trajectories must cross each other at $s=0$. This further leads to $\gamma(0)=-\bar{\gamma}(0)$. Then $K_{tot}(s, u)=0$ at $s=0$. This total amplitude does not have a Regge behavior at $s=0$.

In this paper, we first evaluated exactly the coefficients of the $u^{\alpha(s)}$, $u^{\alpha(s)-1}$ and $u^{\alpha(0)-1}$ terms. Using these coefficients we have shown that an attempt to eliminate the fixed power term by another Regge pole leads to a vanishing total amplitude at $s=0$.

REFERENCES

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