Lorentz-squeezed Hadrons and Hadronic Temperature

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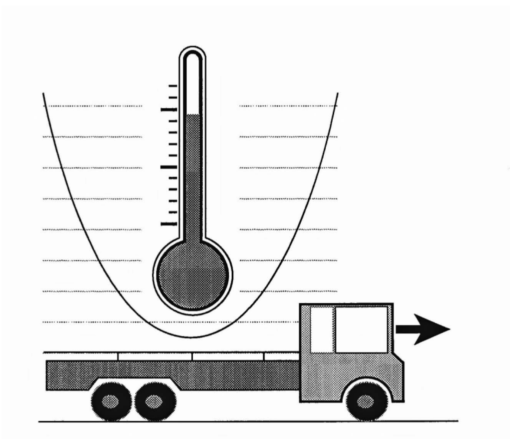
Abstract

It is shown possible to define the temperature of Lorentz-squeezed hadrons in terms of their speed. Within the framework of the covariant harmonic oscillator formalism which is the simplest scientific language for Lorentz-squeezed hadrons, the hadronic temperature is measured through

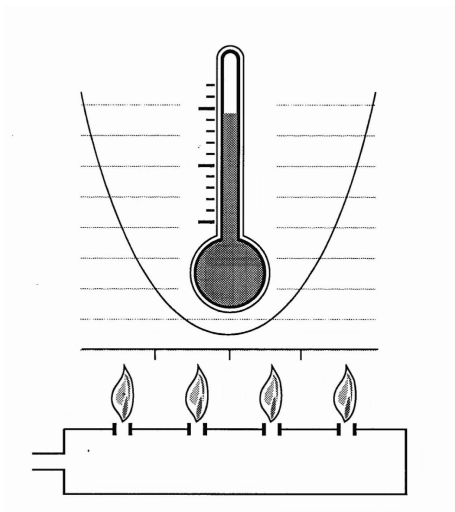
$$\left(\frac{v}{c}\right)^2 = \exp\left(\frac{-\hbar\omega}{kT}\right).$$

As the temperature rises, the hadron goes through a transition from the confinement phase to a plasma phase.



Density matrix from the entangled space-time and the time-separation variable hidden in Feynman's rest of the universe:

$$\rho_\eta(z, z') = \left(1 - (v/c)^2\right) \sum_n (v/c)^{2n} \phi_n(z)\phi_n(z').$$



Density matrix from the thermal excitation:

$$\rho_T(z, z') = \left(1 - e^{-\hbar\omega/kT}\right) \sum_n e^{-n\hbar\omega/kT} \phi_n(z)\phi_n^*(z'),$$

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The purpose of this paper is to introduce the concept of temperature for relativistic extended hadrons, based on the theoretical frameworks of three seemingly different branches of physics, *ie.*, two-mode squeezed states of light citepaper101, thermo-field-dynamics [2], and the space-time geometry of extended hadrons [3]. This concept is consistent with the expectation that rising hadronic temperature leads to a transition from the confinement to the plasma phase [4].

One of the remarkable features of squeezed light is that its underlying scientific language is that of the (2 + 1)-dimensional Lorentz group [1, 5]. Therefore, the study of squeezed states of light may teach new lessons on Lorentz transformations, as in the case of the Thomas precession [6]. In this paper, we would like to point out first that the mathematics of two-mode squeezed states is the same as that for the covariant harmonic oscillator formalism for relativistic extended hadrons [3]. We shall therefore show that fast-moving hadrons undergo a Lorentz squeeze. The squeeze parameter in this case is related to the Lorentz-boost parameter.

Another remarkable feature of two-mode squeezed states of light is that its formalism is identical to that of thermo-field-dynamics [2] and the C^*-algebra [7] which gives a pure-state description for thermally excited bosonic states, as was discussed recently by several authors [8]. The temperature is in this case related to the squeeze parameter. It is therefore possible to define the temperature of a Lorentz-squeezed hadron within the framework of the covariant harmonic oscillator model.,

As is well known, the mathematics of photon-number states is based on the one-dimensional harmonic oscillator. Let us start with the one-dimensional harmonic oscillator described by ,

$$\frac{1}{2}\left[m\omega^2 X^2 - \frac{1}{m}\left(\frac{\partial}{\partial X}\right)^2\right]\Psi_n(X) = (n+1)\omega\Psi_n(X). \tag{1}$$

Under the scale transformation $x = \sqrt{m\omega}X$, this equation becomes independent of ω,

$$\frac{1}{2}\left[x^2 - \left(\frac{\partial}{\partial x}\right)^2\right]\psi_n(x) = (n+1)\psi_n(x). \tag{2}$$

Thus the frequency dependence is solely in the scale transformation from X to x. For the case of photons, $\psi_n(x)$ represents the n-photon state. With this understanding, we can construct a product of ground states of two oscillators with different frequencies ω_1 and ω_2, and thus two different coordinates x_1 and x_2,

$$\psi\left(x_1, x_2\right) = \frac{1}{\sqrt{\pi}}\exp\left[-\frac{1}{2}\left(x_1^2 + x_2^2\right)\right]. \tag{3}$$

In the language of photons, the above wave function corresponds to the zero-photon state in the two-photon mode [1, 8]. This ground state is invariant under rotations around the origin of the two-dimensional space of x_1 and x_2. In particular, we can consider the variables $y_1 = \left(x_1 + x_2\right)/\sqrt{2}$ and $y_2 = \left(x_1 - x_2\right)/\sqrt{2}$. Then the squeezed vacuum corresponds to the contraction and expansion along the y_1 and y_2 axes respectively: ,

$$\psi_\eta\left(x_1, x_2\right) = \frac{1}{\sqrt{\pi}}\exp\left\{-\frac{1}{2}\left[e^{-2\eta}\left(x_1 + x_2\right)^2 + e^{2\eta}\left(x_1 - x_2\right)^2\right]\right\}. \tag{4}$$

In terms of $\psi_n(x)$, this form can be expanded as [8]

$$\psi_\eta (x_1, x_2) = \frac{1}{\cosh\eta} \Sigma_n (\tanh\eta)^n \psi_n (x_1)\, \psi_n (x_2)\,. \tag{5}$$

In the above expression, $\psi_n (x_1)$ and $\psi_n (x_1)$ represent the n-photon states for the first and second modes respectively. This form is identical to the thermally excited field [8, 8]:,

$$\psi_T (x_1, x_2) = \left(1 - e^{-\omega/T}\right)^{1/2} \sum_n e^{-n\omega/2T} \psi_n (x_1)\, \psi_n (x_2)\,, \tag{6}$$

if we identify $(\tanh\eta)^2$ with the Boltzmann factor $\exp(-7\omega/T)$, where T is measured in units of Boltzmann's constant k. In thermo-field-dynamics, x is called the primary coordinate if x_1 is chosen to be 7w 1, and x is called the shadow coordinate [2].

It is now possible to construct the density matrix for the thermally excited state of the first mode by integrating over the shadow coordinate the expression $\psi(x_1, x_2)\, \psi^* (x_1, x_2)$ [8],

$$\begin{aligned} \rho_T (x_1, x_1') &= \int \psi_\eta (x_1, x_2)\, \psi_\eta^* (x_1', x_2)\, dx_2 \\ &= \left(1 - e^{-\omega/T}\right) \sum_n e^{-n\omega/T} \psi_n (x_1)\, \psi_n^* (x_1')\,, \end{aligned} \tag{7}$$

This form of the density matrix for the one-dimensional harmonic oscillator is readily available in the literature [9]. ,

The transformation from the two-oscillator ground state to the thermally excited state had been known in condensed matter physics before the concept of squeezed states of light was introduced. This is known as the Bogoliubov transformation [2, 7, 8].

In this paper, we would like to show that the relativistic quark model can also be framed into the formalism of the Bogoliubov transformation. The relativistic quark model based on the covariant harmonic oscillator formalism is consistent with the observed hadron mass spectrum, hadronic form factors, and Feynman's original form of the parton picture [3, 10, 11]. It also constitutes a representation of the Poincaré group [3, 12].

If the space-time position of two quarks are specified by x_a and x_b respectively, the system can be described by the variables:,

$$X = (x_a + x_b)/2, \qquad x = (x_a - x_b)/2\sqrt{2}. \tag{8}$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks. In the convention of Feynman 3et al1. [13], the internal motion of the quarks can be described by the Lorentz-invariant oscillator equation,

$$\frac{1}{2}\left[\Omega^2 x_\mu^2 - \left(\frac{\partial}{\partial x_\mu}\right)^2\right] \psi(x) = (\omega\lambda)\psi(x), \tag{9}$$

where we use the space-favored metric: $x^\mu = (x, y, z, t)$. The four-dimensional covariant oscillator wave functions are Hermite polynomials multiplied by a Gaussian factor, which dictates the localization property of the wave function. As Dirac suggested [14], the Gaussian factor takes the form,

$$\left(\frac{\Omega}{\pi}\right) \exp\left[-\frac{1}{2}\Omega\left(x^2 + y^2 + z^2 + t^2\right)\right]. \tag{10}$$

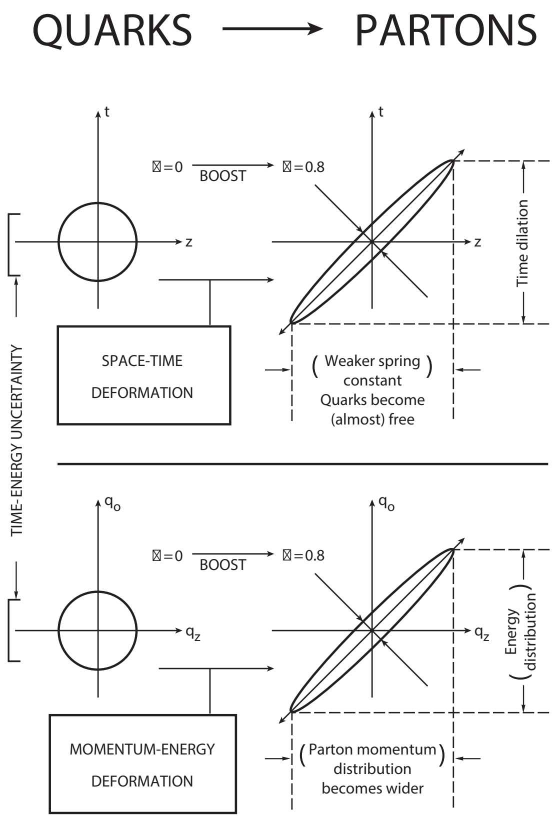

Figure 1: Lorentz-squeezed hadron. The upper half of this figure describes the space-time squeeze, while the lower half is for the squeeze in the momentum-energy plane, where q_z and q_0 measure the longitudinal and time-like momentum separations respectively. This squeeze property, together with the present figure, has been repeatedly discussed in the literature [3, 11]. When the hadron is at rest, it appears as a bound state of quarks. When it moves with its velocity close to that of light, it appears as a collection of free partons. It is interesting to note that this transition mechanism is the same as that of thermally excited states of light from two-mode squeezed states.

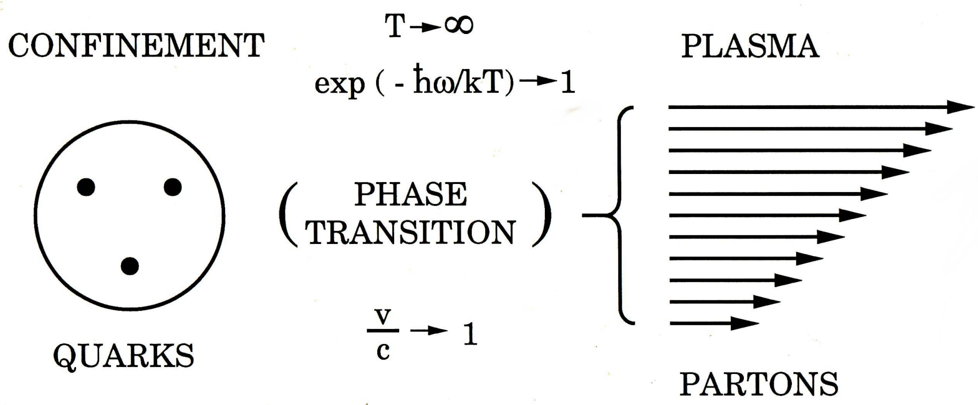

Figure 2: Transition from the confinement phase to a plasma phase. This figure is a new interpretation of Fig. 1.

This expression is not Lorentz-invariant, and its localization undergoes a Lorentz deformation [3, 11]. Since the x and y components are invariant under Lorentz boosts along the z direction, and since the oscillator wave functions are separable in the Cartesian coordinate system, we can drop the x and y variables from the above expression, and restore them whenever necessary.

The Lorentz boost along the z direction takes a simple form in the 1light-cone coordinate system, in which the variables $(z+t)/\sqrt{2}$ and $(z-t)/\sqrt{2}$ are transformed to $e^{\eta}(z+t)/\sqrt{2}$ and $e^{-\eta}(z-t)/\sqrt{2}$ respectively, where η is the boost parameter and is $\tanh^{-1}(v/c)$. Then the ground-state wave function will be Lorentz-squeezed as [3]

$$\left(\frac{\Omega}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{4}\Omega\left[(z+t)^2 + (z-t)^2\right]\right\}. \tag{11}$$

This squeeze property is illustrated in Fig. 1. The above wave function can be expanded as [3]

$$\psi_{\eta}(z,t) = \frac{1}{\cosh\eta}\sum_{n}(\tanh\eta)^{n}\psi_{n}\left(\sqrt{\Omega}z\right)\psi_{n}\left(\sqrt{\Omega}t\right). \tag{12}$$

Indeed, this expression is the same as that for the two-mode squeezed state given in Eq.(5), if η is identified as the squeeze parameter of Eq.(4) or Eq.(5). It is thus possible to relate the boost parameter to the Boltzmann factor by $(tanh\eta)^2 = \exp\left(-\omega/T\right)$, with $\omega = \Omega/m$ where m is the reduced mass of the quark. The hadronic temperature T can therefore be defined as ,

$$\left(\frac{v}{c}\right)^2 = \exp\left(\frac{-\Omega}{mT}\right), \quad T = \frac{\Omega/m}{\ln\left[1 + (M/P)^2\right]}, \tag{13}$$

where M and P are the hadronic mass and its magnitude of momentum respectively. If the hadron is at rest with $P = 0$, T vanishes. The temperature rises as the hadronic momentum increases. As the momentum becomes very large, T increases as $\left(\Omega/mM^2\right)P^2$.

In the present case, the concept of hadronic temperature is derived from the Lorentz-squeezed wave function, and the temperature is a measure of squeeze as in the case of two-mode squeezed states of light [8]. The Lorentz-squeeze of hadronic distribution is observed experimentally in high energy laboratories in the form of the parton model [10, 11]. If we regard z and t as the primary and shadow coordinates respectively, then the hadronic distribution can be derived from the density matrix:

$$\begin{aligned}\rho\left(z,z'\right) &= \int \psi_\eta\left(\sqrt{\Omega}z,\sqrt{\Omega}t\right)\psi^*_\eta\left(\sqrt{\Omega}z',\sqrt{\Omega}t\right)dt \\ &= \left(\frac{1}{\cosh\eta}\right)^2\sum_n (\tanh\eta)^n\psi_n\left(\sqrt{\Omega}z\right)\psi^*_n\left(\sqrt{\Omega}z'\right),\end{aligned} \tag{14}$$

whose diagonal elements become the distribution of the quarks:

$$\rho(z) = e^{-\eta}\left(\frac{\Omega}{\pi}\right)^{1/2}\exp\left[-\left(\Omega e^{-2\eta}\right)z^2\right]. \tag{15}$$

This leads to a wide-spread distribution along the z axis as $(v/c)^2$ becomes close to 1 [3, 11], or equivalently as the temperature T becomes very high. As is indicated in Fig. 1, the momentum distribution undergoes a similar deformation. The simultaneous expansion in both spatial and momentum distributions leads to the transition from the quark model to Feynman's original form of the parton model, in which the hadron appears as a collection of an infinite number of free partons with a wide-spread momentum distribution [**?**, 10, 11]. This means that the rapidly-moving hadron is in a plasma phase.

The Boltzmann factor $\exp(-E/T)$ and $(v/c)^2$ in special relativity are two of the most fundamental quantities in physics. We realize that it is a serious venture to suggest that they are in any way equal to each other, and therefore that an entirely new physical interpretation may emerge from this suggestion. While we are not able to explain fully the significance of this possibility, we are reporting the fact that Tused in this paper is clearly a Lorentz-transformation parameter, while at the same time it is temperature in the thermally excited oscillator [9], as well as in thermo-field-dynamics and squeezed states of light [2, 8]. As is illustrated in Fig. 2, this definition of temperature leads to the conclusion that hadrons undergo a phase transition from the confinement phase to a plasma phase as the temperature rises [4].

We are then led to the question of how to determine the critical temperature at which the system undergoes the phase transition. The parton model does not specify the critical speed at which the hadron becomes a collection of plasma-like partons. This transition is known to be a gradual process. On the other hand, as is illustrated in Fig. 3, the $(v/c)^2$ factor as a function of T has an abrupt change in slope in the interval between $T = 5\Omega/m$ and $15\Omega/m$. The critical temperature is within this interval.

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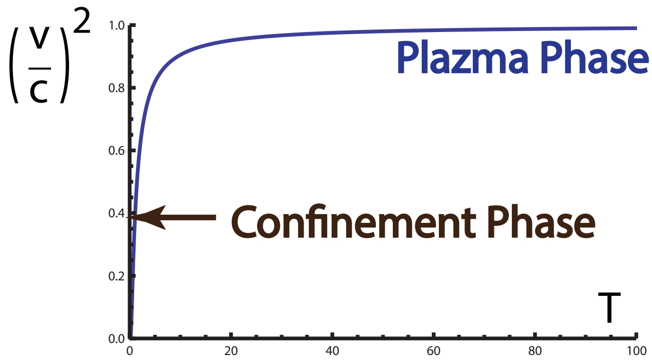

Figure 3: The $(v/c)^2$ factor as a function of the temperature T measured , in units of Ω/m. There is an abrupt change in slope within the interval , 5 ¡ T ¡ 10. The critical temperature is within this interval.

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