Kant and Hegel in Physics

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Abstract

Kant and Hegel are among the philosophers who are guiding the way in which we reason these days. It is thus of interest to see how physical theories have been developed along the line of Kant and Hegel. Einstein became interested in how things appear to moving observers. Quantum mechanics is also an observer-dependent science. The question then is whether quantum mechanics and relativity can be synthesized into one science. The present form of quantum field theory is a case in point. This theory however is based on the algorithm of the scattering matrix where all participating particles are free in the remote past and in the remote future. We thus need, in addition, a Lorentz-covariant theory of bound state which will address the question of how the hydrogen atom would look to moving observers. The question is then whether this Lorentz-covariant theory of bound states can be synthesized with the field theory into a Lorentz-covariant quantum mechanics. This article reviews the progress made along this line. This integrated Kant-Hegel process is illustrated in terms of the way in which Americans practice their democracy.

1 Introduction

Let us look at a Coca-Cola can. It is a circle if we see it from its top, and its side view is a rectangle, as is illustrated in Fig. 1. How would it then appear to a moving observer. This is of course an Einsteinian problem. Einstein studied the philosophy of Kant during his high school years. It is thus natural for him to ask this question.

Niels Bohr was interested in the electron orbit of the hydrogen atom. It is well known how his efforts led to the present form of quantum mechanics. The wave function of the hydrogen atom consists of the rotation-invariant radial function and the angular function consisting of spherical harmonics and spinors. The angular function tells how the orbit looks to observers looking at different angles. This is called the rotational symmetry in physics. The question is how this atom would appear to observers in motion.
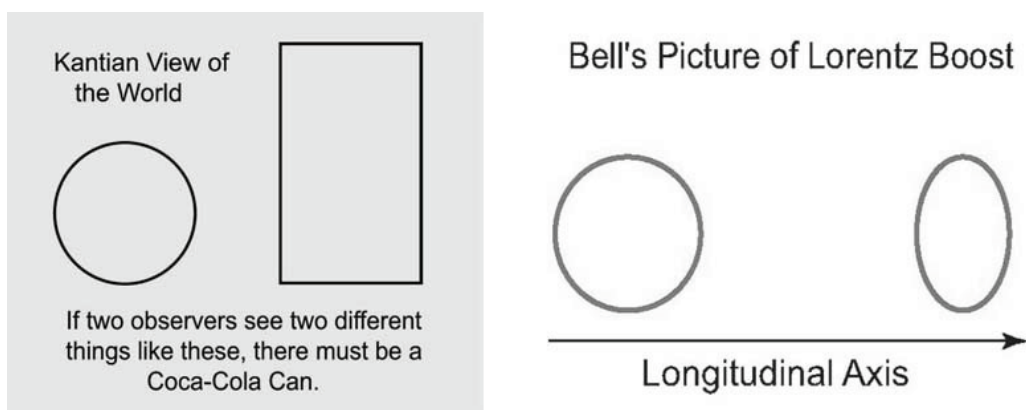

Figure 1: A Coca-Cola can appears differently to two observers from two different angles. Likewise, the electron orbit in the hydrogen atom should appear differently to two observers moving with two different speeds. The elliptic deformation of the circular orbit is from J.S. Bells book [1]. This is deformation is only a speculation based on Einsteins length contraction and does not have any scientific merits. The purpose of the present paper is to clarify this issue.

This is a Kantian question. The observer is in a different environment. Einstein formulated this problem with a mathematics called the Lorentz group. His Lorentzian system consists of three rotations around three different directions in the $(x,\ y,\ z)$ space, three Lorentz boosts in those directions, and three translations along these different directions, plus one translation along the time. There are thus ten operations in the Lorentzian system. three translations along the three different directions. The

mathematics governing these ten operations is called the inhomogeneous Lorentz group Wigner 1939]. The main purpose of this paper is to examine how the hydrogen appears to moving observers in terms of the way Kant and Hegel suggested. We can then ask whether quantum mechanics and special relativity can be synthesized.

In Sec. 2, we examine how the idea of Kant and that of Hegel can be integrated into a single Kant-Hegel procedure in physics. In Sec. 4, we review the attempts made in the past to synthesize quantum mechanics and special relativity. It is noted that the present form of quantum field theory can only deal with scattering problems. It is noted also that Paul A. M. Dirac made his life-long efforts to construct bound-state wave functions that can be Lorentz-boosted. By integrating those efforts. It is shown possible to construct harmonic-oscillator wave functions that can be Lorentz-boosted.

In Sec. 4, we examine whether this Lorentz-transformable wave function can explain what we see in the real world. Let us pick a proton which is a bound state of more fundamental particles called quarks [2]. When it moves with a speed close to that of light, it appears as a collection of free particles called partons [4]. Why does the same proton appear differently? This is precisely Einsteins Kantian question. After settling the issue of bound states in the Lorentzian system. We are led to the question of whether quantum mechanics and Einsteins special relativity can be derived from the same set of mathematical formulas. It is noted that Dirac in 1963 started with two harmonic oscillators satisfying the Heisenberg uncertainty brackets [3]. He then noted that the symmetry from these two oscillators is like that of the Lorentz transformations applicable the five dimensional space with three space coordinate and two time-like coordinates.

In Sec. 5, it is shown that the second time variable in this five dimensional space can be transformed to the translations along the three space-like directions and one time like direction, just like Einsteins Lorentzian system. Indeed, quantum mechanics and special relativity can be derived from the same set of equations, namely the Heisenberg brackets.

Kant and Hegel developed their theories based on the human societies and histories, not on physical theories. It is thus easier to illustrate the integrated Kant-Hegel mechanism in terms of history. The history is the United States is short and transparent. In the Appendix, we shall examine the role of Kant and Hegel while Americans practice their democracy.

2   Integration of Kant and Hegel

As is indicated in Fig. 2, Immanuel Kant and Georg Wilhelm Friedrich Hegel are among the most respected philosophers. Yet, their books are very difficult to read. The best way to understand their ways of reasoning is to construct illustrations.

According to Kant, many things should become one, the ding-an-sich. They just look differently depending on the observers environment and state of mind. According to Hegel, we can create a new wonderful world by synthesizing two different traditions. His philosophy was based on the history. He realized that Christianity is a synthesis of Jewish ethics and Greek philosophy. How can we integrate Kant and Hegel? Kant wanted to derive one from many. Hegel wanted to derive one from two.

Thus, we need a mechanism which will lead many to two, between Kant and Hegel. This way of thinking was developed by ancient Chinese. After the ice age, many people with different backgrounds came to the banks to Chinas northern river. They drew pictures to communicate, and this led to Chinese characters. In order to express their feelings, they sang songs. This is the reason why there are tones in spoken Chinese. How about different ideas? They realized they cannot be united to one. They thus divided them into two opposing groups, namely Yang (plus) and Ying (minus). This way is known as Taoism.

Kant was born in East Prussia (now Kaliningrad, Russia), and spent 80 years of his entire life there. Thus, his way of thinking was framed by what he saw every day. His area was a maritime commercial hub of the Baltic Sea, just like Venice in the Mediterranean world. Many people came to Kants place with many different points of view for the same thing. Thus, Kantianism and Taoism were developed in the same way, as illustrated in Fig. 3. Kant wanted one, but Chinese had to settle with two. Thus, Taoism can stand between Kant and Hegel. We can thus integrate Kant and Hegel by placing Taoism between them. We can illustrate this integrated Kant-Hegel system in terms of the history of the United States. Europeans with different backgrounds came to the new land and settled down in many different areas. They then set up their own government. In order to develop their laws and national policies, they developed two different political parties. These two parties produce the laws and policies applicable to all citizens. This American system is admired by many people of the world. This is an integrated Kant-Hegel system, as is illustrated in Fig. 4.

In the past, the physical laws were developed according to this integrated Kant-Hegel process. Physicists like to unite many different events into one formula or one set of formulas. There are many heavenly bodies. They can be divided into two groups, namely comets (with open orbits) and planets with (localized orbits). Isaac Newton synthesised these two groups into one with his second-order differential equation. This is a Hegelian process. James Clerk Maxwell synthesised the equations governing electricity and those for magnetism into a set of four equations. This created the present-day wireless civilization. As is well known, the present form of quantum mechanics is a synthesis of particle nature and wave nature of matter. Einsteins special relativity synthesizes massive and massless particles. All these are the integrated processes of Kantianism and Hegelianism. The remaining problem is whether quantum mechanics and Einsteins relativity can be synthesised. In this paper, we restrict ourselves to his

Kant (1724-1804) in Kaliningrad (Koenigsberg until 1945),
and Hegel (1770-1831) in Berlin. Photos by Y.S.Kim

Figure 2: Kant and Hegel. Einstein had a very strong Kantian background from his high school years. Yet, he became a great Hegelian synthesizer as a physicist. His photo-electric effect synthesized the particle and wave natures of matter. He synthesized the energy-momentum relations for massive and massless particles. This work is commonly known as Einsteins $E = mc^2$.
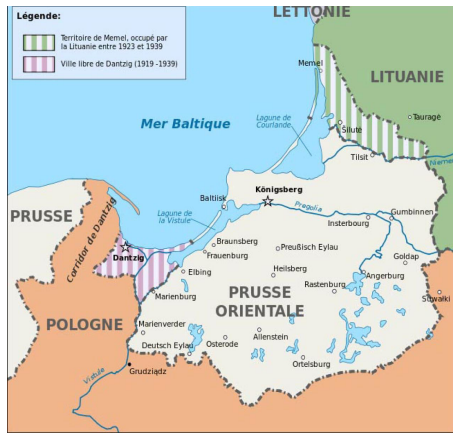
East Prussia before1946, and China (1600-1000 BC).

drawings by Schwarz and Weiss, and by Yu Nenjie.

Figure 3: Geography of Kants East Prussia and Geography of ancient China. Different groups with their own ways of thinking come to the same area, Their views toward the same thing could be different.
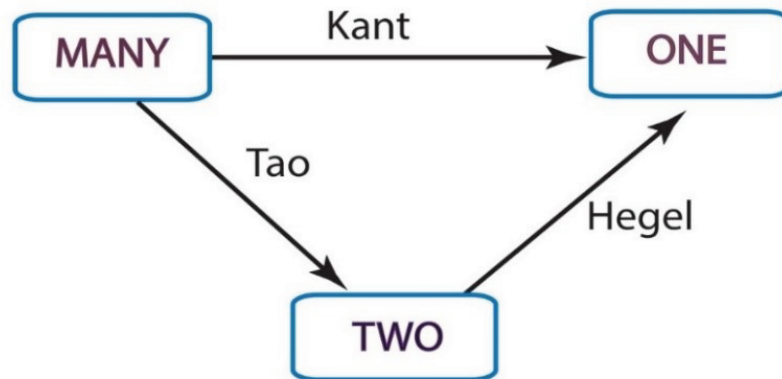

Figure 4: Integration of Kantianism and Hegelianism. Kant wanted to reduce many-to-one while Hegel wanted to combine two-into-one. Thus, we need many-to-two mechanism in between. This is what Taoism is about.

special relativity, even though his general relativity receives more public attention these days. Einsteins nickname is still $E = mc^2$, which was a product of his special theory of relativity.

3 Synthesis of of Quantum Mechanics amd Special Relativity

The present form of quantum mechanics was developed for the Galilean system. While the Galilean system operates with three translations and three-rotations on the space of (x, y, z), the time variable does not interfere with the coordinate transformations. However, was stated in Sec. 1, the space and time of Einsteins special relativity is based on the Lorentzian system. This system operates in the four-dimensional space of (x, y, z). In this Lorentzian system, there are rotations in the three-dimensional space of (x, y, z). In addition, when the observer moves with a constant speed, the time variable comes in. We call the observers velocity change "Lorentz boost." The boost can be made in three different directions. We call this symmetry system Lorentz covariance. In additional, there are four translations along the four-dimensional space. Let us call the system of these three rotations, three boosts, and four translation the Lorentzian system.

The difference between the Galilean system and the Lorentzian system is spelled out in Table 1. Quantum mechanics was originally developed in the Galilean system, but it was a great challenge during the 20th Century to construct quantum mechanics in the Lorentzian space and time.

Quantum field theory is a case in point. The mathematical algorithm of this theory is based on the scattering matrix where all participating particles are free in the remote past and free in the remote future. For making computations of the scattering matrix, Feynman diagrams provide mathematical transparencies with excellent physical interpretations. How about bound-state particles? The particles are not free in the remote past and remote future.

As indicated in Fig. 5, our understanding of bound states and scattering states did not go together all the time. As Feynman suggested [5], while Feynman diagrams are useful for running waves, we can use harmonic oscillators to understand quantum bound states in the Lorentzian system. In their paper of 1971, however, Feynman *et al.* did not do a very good job in constructing the harmonic oscillators in the Lorentzian world. Their oscillator wave functions increase as the time separation variable become large. Thus, their wave functions are meaningless in quantum mechanics Indeed, before Feynman *et al.*, Dirac attempted to construct a representation of the Lorentz group using harmonic oscillator wave functions [6]. Before 1971, a number of authors wrote down Lorentz-covariant oscillator wave functions [7, 8, 9].

Table 1: Three different systems. In the traditional Galilean system, there are three rotational and three translational degrees of freedom. In the Lorentzian system, four boost operations are possible and one additional translation, namely along the time direction. In 1963, Dirac constructed a space-time symmetry from two harmonic oscillators [3]. This will be discussed later in this paper.

Systems	Galilean	Lorentzian	Two Oscillators
Rotations	$J_x, J_y, J_z.$	$J_x, J_y, J_z.$	$J_x, J_y, J_z.$
Boosts	None	$K_x, K_y, K_z.$	$K_x, K_y, K_z.$
Translations	$P_x, P_y, P_z.$	$P_x, P_y, P_z, P_t.$	Q_x, Q_y, Q_z, S_0 contracted to $P_x, P_y, P_z, P_t.$

Yet, Feynman *et al.* ignored them all. Since 1973, mostly with Marilyn Noz, the present author started publishing papers on this subject [10] and continued writing papers and books along the same line [11, 12, 14, 15, 18, 19]. With those papers, it is now possible to integrate Diracs lifelong efforts to construct Lorentz-covariant oscillator wave functions. Dirac published three papers toward the Lorentz-covariant oscillators. In 1927, Dirac said that the c-number time-energy uncertainty should be included in Einsteins Lorentzian world [22]. In 1945, he suggested harmonic oscillators for a representation of the Lorentz group [6]. In 1949, he introduced the light-cone coordinate system for Lorentz boosts, saying that the Lorentz boost is a squeeze transformation dir49. Diracs papers are like poems, but they contain no diagrams. Thus, we can use diagrams to accomplish what Dirac did not do, that is to integrate his own papers. As is illustrated in Fig. 6, his three papers can be integrated into an ellipse as a squeezed circle tangent to Einsteins hyperbola.

Let us go back to Fig. 5, it is important to note that both the Feynman diagrams and the oscillator formalism given in this section can be constructed from the same set of commutation relations, which is known as the Lie algebra of the inhomogeneous Lorentz group [11]. It is also important note that the Feynman diagrams and

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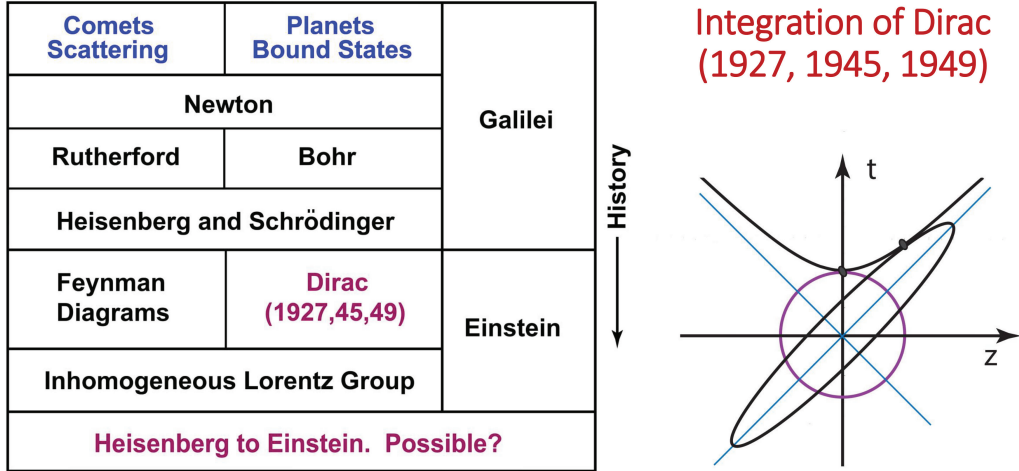| Comets Scattering	Planets Bound States	
Newton		Galilei
Rutherford	Bohr	
Heisenberg and Schrödinger		
Feynman Diagrams	Dirac (1927,45,49)	Einstein
Inhomogeneous Lorentz Group		
Heisenberg to Einstein. Possible?		

Integration of Dirac (1927, 1945, 1949)

Figure 5: Historically, our unified understanding of open and closed orbits has been very brief. Quantum field theory and covariant oscillator lead to quite different mathematical formulas. However, they both are representations of the inhomogeneous Lorentz group [11].

the Lorentz-covariant oscillator wave functions are constructed from the same set of physical principles governing quantum mechanics and special relativity [20].

4   Quark-parton Puzzle

On hundred years ago, Bohr and Einstein met occasionally to discuss physics. Bohr was worrying about the electron orbit of the hydrogen atom, while Einsteins main interest was how things appear to moving observers. Thus, they could have talked about how the hydrogen atom looks to a moving observer. However, there are no records indicating that they ever talked about this issue. If they did not, they are excused. There were and still are no hydrogen atoms moving with relativistic speeds. Since the total charge of the hydrogen atom is zero, it cannot be accelerated even these days.

On the other hand, modern particle accelerators routinely produce many protons moving with the speed very close to that of light. These protons are not hydrogen atoms. However, they are also bound states within the same framework of quantum mechanics. As indicated in Fig. 7, it is possible to study moving hydrogen atoms by looking at moving protons. Indeed, according to the quark model [2], the proton is a quantum bound state of three quarks. Then the question is how the proton appears when it moves fast. In 1969, Feynman observed that the proton looks quite differently
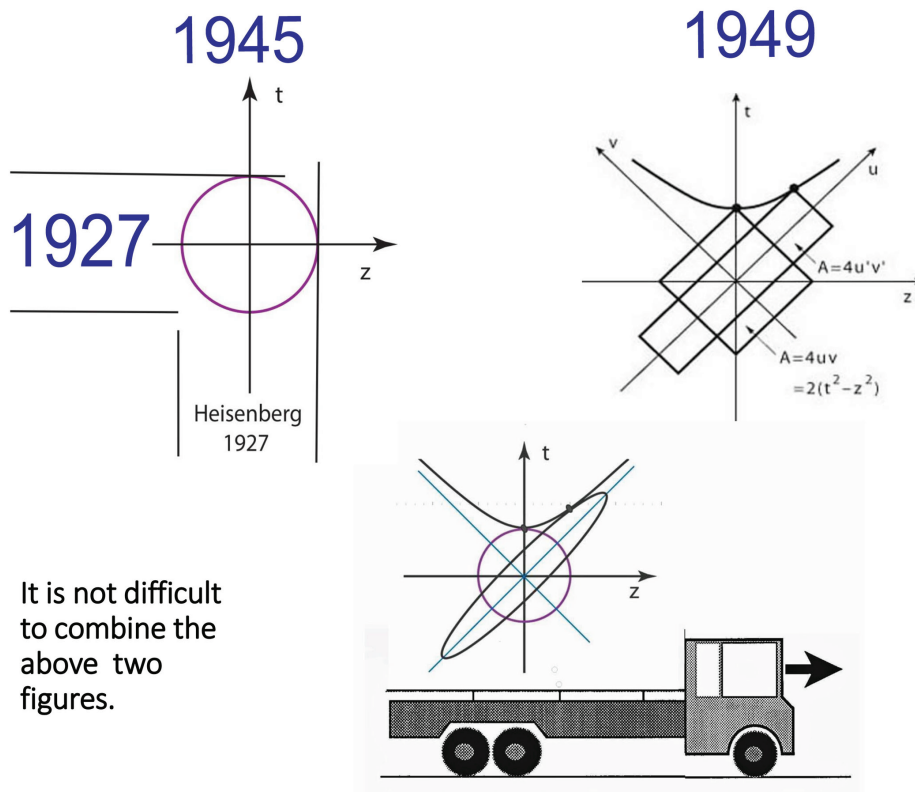
Figure 6: Translation of Diracs papers into pictorial language. The synthesis of his 1927 and 1945 result in a circle. His squeeze transformation of 1949 leads to a squeezed circle or ellipse shown in this figure.

100 years ago, Bohr was worrying about the orbit of the hydrogen atom.

Einstein was interested in how things look to moving observers. Then how the hydrogen atom would look to moving observers? This was a metaphysical question for them.

50 years ago, the proton became a bound state of the quarks sharing the same quantum mechanics as that for the hydrogen atom, according to Gell-Mann. If it moves with a speed close to that of light, the proton appears as a collection of partons, according to Feynman.

Question. Does the proton appear like a collection of Feynman's partons to a moving observer?
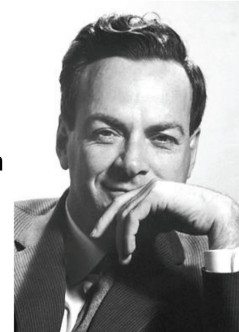
Photo of Gell-Mann by Y.S.Kim (2010), all other photos are from the public domain.

Figure 7: The Bohr-Einstein on the hydrogen, and Gell-Mann and Feynman on the proton. Both the hydrogen and proton are bound states sharing the same quantum mechanics. The proton can be accelerated, while this is not the case for hydrogen atom.
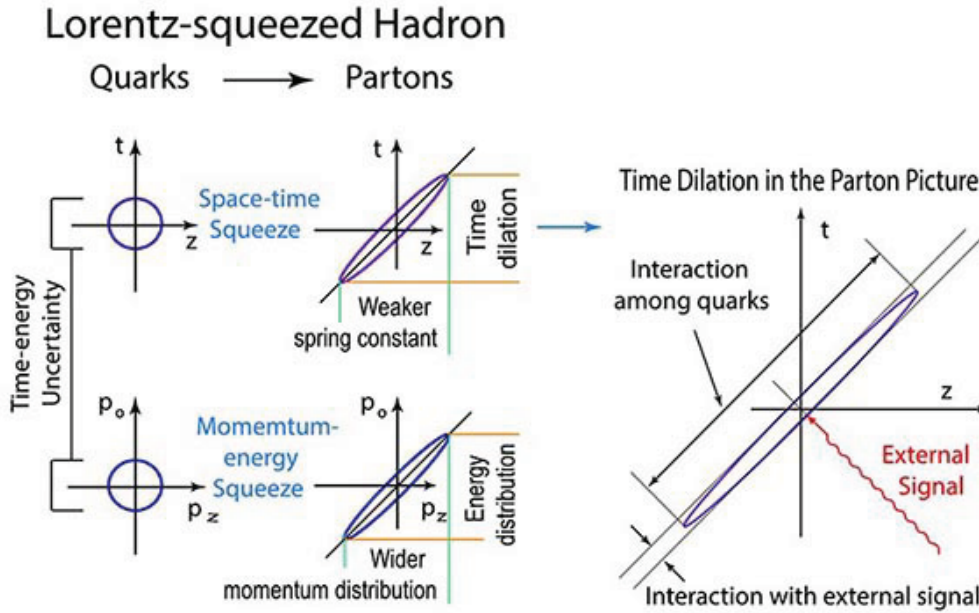
Figure 8: Space-time and momentum energy wave functions. As the hadronic speed reaches that of light, both wave functions become concentrated along one of the light cones.

when it moves with ultra-fast speed [4]. It appears like a collection of light-like particles. Feynman called them partons. These partons have the following peculiar properties.

a. Feynmans parton picture is valid only for protons moving with velocity close to that of light.

b. The interaction time between the quarks becomes dilated, and partons behave like free independent particles.

c. The momentum distribution of partons becomes widespread as the proton speed increases.

d. The number of partons seems to be infinite or much larger than that of the constituent quarks.

The question is whether it is possible to explain Feynmans parton picture within the framework of quantum mechanics and special relativity. In order to answer this question, we need a bound-state wave function which can be Lorentz-boosted. In Sec. , we constructed the Gaussian function that can be Lorentz-boosted. The Gaussian function is different from the wave function for the hydrogen atom, but both wave functions share the same quantum mechanics. In 1964, Gell-Mann proposed the quark

model for hadrons. The hadrons are bound states of the quarks, and their mass spectra are like those of the harmonic oscillators [5]. The proton is a hadron and is a bound state of three quarks. In the oscillator regime, the wave function for this three-body system is a product of two wave functions [5]. It is thus sufficient to study the Lorentz-boost property of the two-quark system which was discussed in Sec. 4. In the oscillator system, the momentum wave function of the Gaussian wave function is also Gaussian, and it becomes Lorentz-squeezed exactly in the same way as in the case of the space-time wave function. Thus, we can extend Fig. 6 to Fig. 8 [12, 15].

Indeed, according to this figure, the quarks become light-like particles with a wide-spread momentum distribution, interacting with an external like free particles. Furthermore, since they are like light-like, the particle number is not constant as in the case of black-body radiation. Thus, this figure provides the answers to all of the puzzles raised on the parton picture listed earlier in this section. As was stated before, the proton wave function is a product of two oscillator wave functions. Using this wave function, Paul Hussar computed the parton distribution function for the proton, and it is in reasonable agreement with the observed parton distribution [24]. Let us go to Table 2. The Lorentz-covariant oscillator, which can be viewed as an integration of Diracs papers, provides a one formula for the proton at rest (quark model) and the ultra-fast proton (parton picture). The second row of this table is about the role of Wigners little group for internal space-time symmetries [25]. This row tells that Wigners little groups explain why the internal space-time symmetries appear differently to moving observers. Again, this is a Kantian problem and was discussed detail in the literature [12, 13, 15, 16, 17].

5 Integration of Quantum Mechanics and Special Relativity

In Secs. 4 and 5, we observed that it is possible to construct harmonic oscillator wave functions that can be Lorentz-boosted. Furthermore, it can settle the one of the Kantian problems we observe in high-energy laboratories producing ultra-fast protons. In addition, we noted that the covariant oscillators and quantum field theory can be constructed within the same Lorentzian system. They also share the same set of physical of physical principles.

In that case, we are led to the question of whether these two scientific disciplines can be derived from the same set of equations. For this purpose, Dirac considered two harmonic oscillators. For the single-oscillator system, we can use the step-up and step-down operators to write down Heisenbergs uncertainty brackets.

For the two-oscillator system, there are four such operators. Dirac constructed ten quadratic forms with those step-up and step-down operators. He then noted that

Table 2: Further contents of Einsteins $E = mc^2$. The first row of this table in well known. The second table is for internal space-time symmetries, the photon spin along the direction of momentum remains invariant, but the perpendicular components become one gauge degree of freedom [13, 16]. The covariant oscillator explains the peculiarities of Feynmans parton picture

	Slow Masssive	between	Ultra fast Massless
Einstein's $E = mc^2$	$E = p^2/2m$	$E = \sqrt{(cp)^2 + m^2 c^4}$	$E = cp$
Wigner's Little Groups	$S_3,$ S_1, S_2	Internal Symmetry	Helicity Gauge Trans.
Integration of Dirac 1927,45,49.	Gell-Mann's Quark Model	Covariant Oscillators	Feyman's Parton Picture

Table 3: Diracs ten quadratic forms which satisfy a closed set of commutation relations identical to that of the Lorentz group applicable to three space-like coordinates and two time-like coordinates. The J operators are for the rotations in the three-dimensional space. The three K operators generate Lorentz boosts along three different directions. This table contains only six of the ten generators.

Dirac Oscillators	Differential
$J_1 = \frac{1}{2} \left(a_1^\dagger a_2 + a_2^\dagger a_1 \right)$	$-i \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$
$J_2 = \frac{1}{2i} \left(a_1^\dagger a_2 - a_2^\dagger a_1 \right)$	$-i \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right)$
$J_3 = \frac{1}{2} \left(a_1^\dagger a_1 - a_2^\dagger a_2 \right)$	$-i \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$
$K_1 = -\frac{1}{4} \left(a_1^\dagger a_1^\dagger + a_1 a_1 - a_2^\dagger a_2^\dagger - a_2 a_2 \right)$	$-i \left(x \frac{\partial}{\partial t} + t \frac{\partial}{\partial x} \right)$
$K_2 = +\frac{i}{4} \left(a_1^\dagger a_1^\dagger - a_1 a_1 + a_2^\dagger a_2^\dagger - a_2 a_2 \right)$	$-i \left(y \frac{\partial}{\partial t} + t \frac{\partial}{\partial y} \right)$
$K_3 = \frac{1}{2} \left(a_1^\dagger a_2^\dagger + a_1 a_2 \right)$	$-i \left(z \frac{\partial}{\partial t} + t \frac{\partial}{\partial z} \right)$

Table 4: Four additional quadratic forms in Diracs paper of 1963. They correspond to the generators operating on the second time variable.

Dirac Ocillators	Differential
$Q_1 = -\frac{i}{4}\left(a_1^\dagger a_1^\dagger - a_1 a_1 - a_2^\dagger a_2^\dagger + a_2 a_2\right)$	$-i\left(x\frac{\partial}{\partial s} + s\frac{\partial}{\partial x}\right)$
$Q_2 = -\frac{1}{4}\left(a_1^\dagger a_1^\dagger + a_1 a_1 + a_2^\dagger a_2^\dagger + a_2 a_2\right)$	$-i\left(y\frac{\partial}{\partial s} + s\frac{\partial}{\partial y}\right)$
$Q_3 = \frac{i}{2}\left(a_1^\dagger a_2^\dagger - a_1 a_2\right)$	$-i\left(z\frac{\partial}{\partial s} + s\frac{\partial}{\partial z}\right)$
$S_0 = \frac{1}{2}\left(a_1^\dagger a_1 + a_2 a_2^\dagger\right)$	$-i\left(t\frac{\partial}{\partial s} - s\frac{\partial}{\partial t}\right)$

they satisfy the closed set of commutation relations which is the same as that for the generators of the Lorentz group applicable to the five-dimensional space consisting of three space coordinates $(x,\ y,\ z)$ and two time coordinates t and s [3].

Table 3 gives three rotation generators and three boost generators with respect to the time variable t, along with corresponding Diracs two-oscillator forms. The operators of this table do not depend on the second time variable s. The J operators there generate rotations in the three-dimensional space $(x,\ y,\ z)$, and the K operators generate Lorentz boosts along those three different directions. Thus, the six operators given in Table generate the Lorentz group familiar to us.

In addition to the six quadratic forms given in Table 4, Dirac constructed four additional quadratic forms. They correspond to the differential operators given in Table 4. The differential operators in Table 4 do not depend on the time variable t, but depend only on the second time variable s. We are now interested in converting the differential forms in this table into four translation generators, using the group contraction procedure introduced first by Inönü and Wigner inonu53. In their original paper, Inönü and Wigner obtained the Galilean system from the Lorentzian system. Since then, this contraction procedure was used for the unification of Wigners little groups for massive and massless particles [13, 15, 16]. This procedure unifies the internal space-time symmetries of massive and massless particles, as Einsteins $E = mc^2$

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Table 5: Contraction of the s-dependent operators. According to Inönü-Wigner procedure for group contractions, we can let s = 1. This procedure transforms the Q and S operators into translation operators.

	Differential	contracted to
Q_1	$-i\left(x\frac{\partial}{\partial s} + s\frac{\partial}{\partial x}\right)$	$-i\frac{\partial}{\partial x}$
Q_2	$-i\left(y\frac{\partial}{\partial s} + s\frac{\partial}{\partial y}\right)$	$-i\frac{\partial}{\partial y}$
Q_3	$-i\left(z\frac{\partial}{\partial s} + s\frac{\partial}{\partial z}\right)$	$-i\frac{\partial}{\partial z}$
S_0	$-i\left(t\frac{\partial}{\partial} - s\frac{\partial}{\partial t}\right)$	$i\frac{\partial}{\partial t}$

does for the energy-momentum relation, as indicated in Table 2. This contraction procedure has been employed for the present purpose of converting all four operators in Table 3 into four translation generators [18, 19]. The result is shown in Table 5.

This contraction procedure tells us to fix the s variable and set $s = 1$ for the generators given in Table 4 [18, 19]. They then become contracted to the translation generators given in Table 5. Indeed, the six generators of the Lorentz group given in Table 1 together with the four translation generators constitute the generators of the inhomogeneous Lorentz group or Einsteins Lorentzian system of space and time. This process is compared with the traditional Galilean and Lorentz systems in Table 1.

Let us go back to Fig. 5. This figure asks whether it is possible to derive Einsteins Lorentzian world from the principles of quantum mechanics. The answer to this question is YES. While the Diracs oscillator algebra is derivable from the Heisenberg brackets, the Heisenberg brackets are also derivable from the oscillator algebra. Thus, both quantum mechanics and special relativity are derivable from the same set of equations. The synthesis of quantum mechanics and special relativity is now complete.

Concluding Remarks

Kant and Hegel are very familiar names to us. They formulated their ideas based on what they observed and what they learned. It is interesting to note that Einstein started as a Kantianist but become a Hegelianist while doing physics. Indeed, physics develops along the integrated Kant-Hegel line. The most pressing task of our time in physics is a Hegelian synthesis of quantum mechanics and theories of relativity. For the single-oscillator system, we can use step-up and step-down operators to write down Heisenbergs uncertainty brackets. For the two-oscillator system, there are four such operators. Dirac constructed ten quadratic forms with those step-up and step-down operators. He then noted that they satisfy the closed set of commutation relations which is the same as that generators of the Lorentz group applicable to the five-dimensional space consisting of three space coordinates (x, y, z) and two time coordinates t and s [3]. In this paper, it is pointed out that those ten generators can be transformed to the ten generators of Einsteins Lorentzian system. Thus, quantum mechanics and special relativity come from the same set of equations.

Anowledgnents

Dirac and Feynman in Poland (1962).   I went to the same place in order show my respect to them (2013).

Photo of Dirac and Feynman by Marek Holzman, from the Caltech Photo Lab.

Figure 9: This photo of Dirac and Feynman appeared on the cover of the Physics Today (August 1963). This photo was taken during the Relativity Conference organized by Leopold Infeld in July of 1962 at the Jablonna Palace near Warsaw, Poland. To show my great respect for both Dirac and Feynman, I went to the same place and produced my own photo. I am 168 cm tall and not short, but both Dirac and Feynman had longer legs.

pleasure of my photo taken at the spot where they spoke to each other. Their photo a lack of communication between them. It has been a great challenge for me to fix the gap between them. Finally, I am indebted to John S. Toll who provided my meeting with Dirac in 1962. He was always helpful to me whenever I whenever needed help throughout my academic career.

Appendix

Traditionally, philosophers wrote their theories based on the religion, history, cultural conflicts. Their theories are quite separate from physical phenomena. Thus, it is much easier to illustrate their philosophies using historical developments.

The history of the United States is short and transparent. After the first journey of Christopher Columbus (1492-93), many Europeans moved to the New Land. In 1776, the Declaration of Independence was ratified. This document was written before Kant and Hegel became prominent, and it does not say anything about political parties.

In practice, these days, the democratic system of the United States is functioning with two political parties. Americans did not construct this system based on any theories of government written before. They constructed this system while practicing their democracy. How did they construct? The country consisted of many different ethnic groups with different cultural backgrounds. They were spread over many different areas in the North American continent. How it is possible to construct a national policy satisfactory to all those citizens?

While practicing democracy, it is necessary to construct one national policy based on all different opinions. Thus, the Kantian process of many-to-one is desirable. However, it is not practical. Therefore, the more practical solution was to place those many opinions into two different groups. It is then possible to "synthesize" two opinions into one, according to the Hegelian synthesis. This process is illustrated in Fig. 4, which was used to illustrate the way in which Einstein developed his theories.

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