University of Maryland

Department of Physics

Physics 275 Notes

Don Schmadel

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1 Analytical Tools

1.1 Types of Errors

Limit the number of calculated digits to be consistant with measured digits

1.1.1 Systematic Errors

Example: correcting 2 meter rule: divide or multiply
Divide or multiply? Double check to get proper value

1.1.2 Quasi Systematic Errors

For analog, error is generally ~1/3 of last division
For digital, error is generally 1/2 last digit
Can be reduced by scanning methods.

1.1.3 Random Errors

Generally change with every measurement.
Error is given by standard deviation of the measurements.
Can be reduce by making several measurements.(see below)
This approach is limited by drift of other quantities, the degradation fo the sample, etc.

Are quantified by the Standard Deviation

$$\sigma_x = \left[\frac{1}{N-1}\sum_{k=1}^{N}(x_k - \langle x\rangle)^2\right]^{1/2}$$

Since the data is usually used to calculate $\langle x\rangle$, then use $\frac{1}{N-1}$ as shown.

1.1.4 Accuracy versus Precision

Accuracy refers to the the degree of closeness of a measurement to that quantity's true value.
Precision refers to the scatter in different measurements of the same quantity, e.g. repeatability.

1.2 Combining Measurements for the same quantity to lower random error.

1.2.1 Weighted Mean

Weighted Mean is used when values of x_n have different uncertanties σ_n:

$$x_{\text{WeightedMean}} = \frac{\sum_{n=1}^{N} \frac{x_n}{\sigma_n^2}}{\sum_{n=1}^{N} \frac{1}{\sigma_n^2}}$$

The uncertainty of the weighted mean is

$$\sigma_{\text{WeightedMean}} = \frac{1}{\sqrt{\sum_{n=1}^{N} \frac{1}{\sigma_n^2}}}$$

1.2.2 Mean (usual form when uncertainties are equal)

The approach above gives the simple average when the uncertainties are the same:

$$x_{\text{Mean}} = \sum_{n=1}^{N} \frac{x_n}{N}$$

Likewise, when the values for σ_n above are all the same and equal to σ, it gives:

$$\sigma_{\text{Mean}} = \frac{\sigma}{\sqrt{n}}$$

1.3 Combining Measurements of different quantities related by an equation to determine error propagation.

1) Produce the the total differential.
2) Square terms so they don't cancel, because in real life they generally don't.
3) The variance (which equals the standard deviaton squared) is the square of the total differential.
   This is illustrated below in “1.3 Finding fitting parameters by minimizing Chi Squared”.

1.4 Distributions

1.4.1 Binomial: how often should something happen?

$$P(n, N) = \frac{N!}{n!\,(N-n)!}\, p^n (1-p)^{N-n}$$

Can plot P as a function of n, then the width or Standard Deviaition is:

$$\sigma = \sqrt{\text{Variance}} = \sqrt{N p (1-p)}$$

EXPLANATION

1) N trials:

N

XXXXXXXXXXXX

2) Probability that the first n are of a certain type and that the remaining type are not of that type:

n N-n

11111 000000000000

p^n $(1-p)^{N-n}$

3) Number of different arrangements:

$$\frac{N!}{n!\,(N-n)!}$$

4) Put it all together:

$$P(n, N) = \frac{N!}{n!\,(N-n)!}\, p^n (1-p)^{N-n}$$

1.4.2 Poisson: Can be derived from Binomial with *N* large but with small *P* so that *N* × *P* finite

For example, N atoms but only small portion $N \times P = \mu$ decay.

$$P_\mu(\nu) = e^{-\mu}\,\frac{\mu^\nu}{\nu!}$$

Where $P_\mu(\nu)$ is the probablity of ν decays

and μ is the expected or average number of decays

The Standard Deviaition is given by $\sqrt{\nu}$

1.5 Chi Squared

1.5.1 Catagorized data

$$\chi^2 = \sum_{k=1}^{N} \frac{(E_k - O_k)^2}{E_k}$$

where N is the number of bins

O_k is observed number in bin k

and E_k is the expected or theoretical number in bin k.

Note1: The number of degrees of freedom is N minus the number of fitting parameters, so you must pick the number to bins to be larger than the number of fitting parameters.

Note 2: When the number of trials is known and all the trials have to be distributed among the bins, then the number of degrees of freedom is further reduced by 1, since after distributing among the first bins, the ramaining must go into the last bin. So, even if there are no fitting parameters the number of degrees of freedom would be $N - 1$.

Example 1: Dice

Example 2: Radioactive Decay

1.5.2 Continuous data

$$\chi^2 = \sum_{k=1}^{N} \frac{(E_k - O_k)^2}{{\sigma_k}^2}$$

Reduced χ^2: simply divide χ^2 by the total number of degrees of freedom.

1.6 $\mathcal{P}(\chi^2, d)$

The p-value is the probability of getting a value of Chi Squared equal or larger than yours assuming that the theory is correct.
You could informally consider it to be the probablilty that the theory is correct based on your data.

1.7 Finding fitting parameters by minimizing Chi Squared

Consider the equation for the acceleration of a particle of mass m caused by an electric field E:

$$m \;\; a = q\,\mathcal{E}$$

or, combining the mass and charge as $\gamma = \frac{m}{q}$

$$\gamma a = \mathcal{E}$$

As we did in the free falling mass experiment, assume that we can measure the location of the particle at various times. Therefore, we need to integrate the equation to get the position as a function of time. This will leave us with additional constants or parameters that we will later determine from our data by minimizing χ^2:

$$\gamma \, \frac{dv}{dt} = \mathcal{E}$$

$$\gamma \, v = \mathcal{E}\, t + c_1$$

$$\gamma \, \frac{dx}{dt} = \mathcal{E}\, t + c_1$$

$$\gamma \, x = \frac{1}{2}\mathcal{E}\, t^2 + c_1\, t + c_2 \qquad \text{EQ. 1}$$

where γ, c_1, c_2 are fitting parameters .

Recall that Chi Squared is given by:

$$\text{Chi Squared} = \sum_{k=1}^{n} \frac{(\text{Expected}_k - \text{Observed}_k)^2}{{\sigma_k}^2} \qquad \text{EQ. 2}$$

To use Chi Squared, we would like to have an expression that separates the observed values from the expected values. To do this we can rewrite EQ. 1 as:

$$0 = -\gamma x + \frac{1}{2}\mathcal{E} t^2 + c_1 t + c_2 \qquad \text{EQ. 3}$$

Our expected value can be considered to be "0", which has no error associated with it. The observed value is:

$$\text{Observed}_k = -\gamma x_k + \frac{1}{2}\mathcal{E} t_k^2 + c_1 t_k + c_2 \qquad \text{EQ. 4}$$

So, EQ. 2 becomes:

$$x^2 = \sum_{k=1}^{n} \frac{\left(0 - \left(-\gamma x_k + \frac{1}{2}\mathcal{E} t_k^2 + c_1 t_k + c_2\right)\right)^2}{\sigma_k}$$

Or simply

$$x^2 = \sum_{k=1}^{n} \frac{\left(-\gamma x_k + \frac{1}{2}\mathcal{E} t_k^2 + c_1 t_k + c_2\right)^2}{\sigma_k} \qquad \text{EQ. 2'}$$

To get σ_k we simply take the total derivative of Observed_k:

$$dO_k = \frac{\partial O_k}{\partial x_k} dx_k + \frac{\partial O_k}{\partial t} dt + \frac{\partial O_k}{\partial \mathcal{E}} d\mathcal{E}$$

$$dO_k = -\gamma\, dx_k + (\mathcal{E} t_k + c_1)\, dt + \frac{1}{2} t^2\, d\mathcal{E}$$

Finally, we square the terms to prevent canceling of errors:

$$\sigma_k^2 = (\gamma)^2 \sigma_x^2 + (\mathcal{E} t_k + c_1)^2 \sigma_t^2 + \left(\frac{1}{2} t^2\right)^2 \sigma_{\mathcal{E}}^2 \qquad \text{EQ. 5}$$

EQ 2' and EQ 5 can then be used in a program like Solver or Parafit which finds values for the fitting parameters that will minimize Chi Squared.