Gravitational-Wave Data Analysis: Lecture 5

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Outline for Today

- **Burst search methods**
- **Coherent burst searches**
- **Searches for stochastic GW**
- **Summary of GW signals and search methods**

Burst Search Philosophy

We're listening to the whole sky – who knows what's out there?

Models are OK, but don't put *too* much faith in them!

Goal: be able to detect *any* signal

… if it has sufficient power within the sensitive frequency band

… and is "short"

Target Signals for GW Burst Searches

Modeled burst searches

Targets:

- Black hole ringdown
- Neutron star ringdown
- Cosmic string cusp
- Parabolic encounter

Use matched filtering

Issues generally similar to binary inspiral searches

Generic burst searches

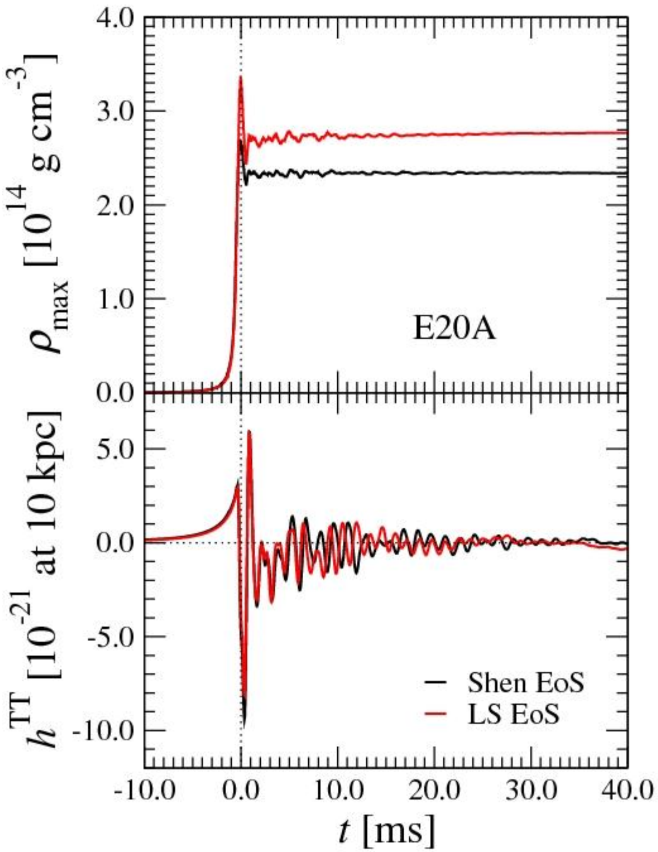
Targets:

- Binary black hole merger
- Core collapse supernova
- Signals deviating from model expectations
- Other unexpected or unmodeled sources

Use robust detection methods that do not rely on having a model of the signal

Some Specific Astrophysical Targets

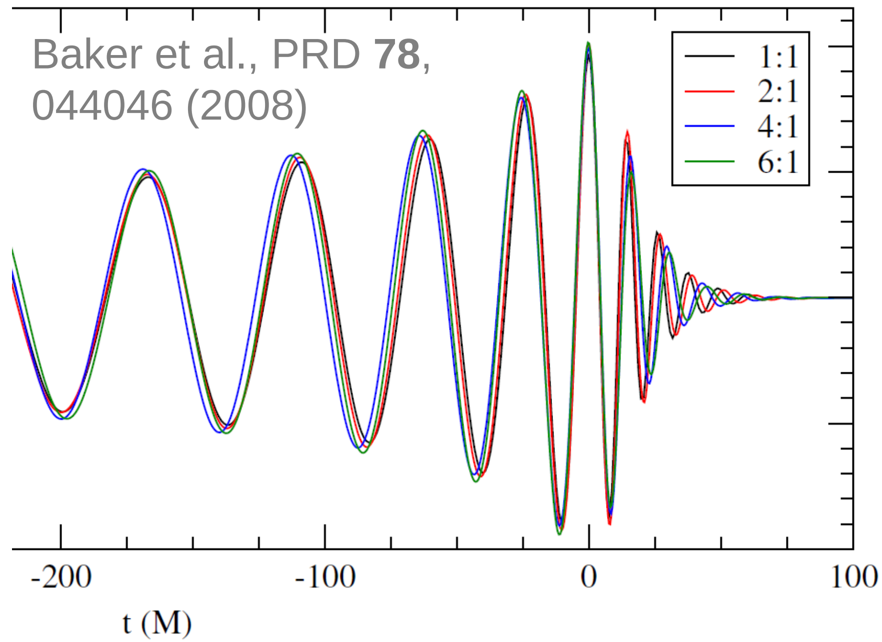
Stellar core collapse



Dimmelmeier et al., PRD **78**, 064056 (2008)

Also see www.stellarcollapse.org

Binary coalescence with in-band merger



Back up the matched-filtering search for rapid inspirals, especially with high mass and/or spins

But also be open to detecting something else…

A Variety of Burst Search Methods

Multiple burst search methods are in active use by LIGO & Virgo

- Mathematical arguments about optimality only go so far
- Implementation details are critical
- Data conditioning, robustness against non-stationary noise, …
- Some degree of competition and cross-pollination

LSC & Virgo have published (or will soon) **many burst searches:**

- "All-sky" burst searches: S1, S2, S3, S4, S5/VSR1, S6/VSR2+3
- GRB burst searches:
  - 39 GRBs during S2/S3/S4, 137 during S5/VSR1, 150 during S6/VSR2+3
  - GRB 030329, GRB 070201, GRB 051103
- Magnetar flare GW burst searches:
  - SGR 1806–20 giant flare QPO search, SGR 1900+14 storm "stack" search
  - GW bursts from flares emitted by six different magnetars
- Cosmic string cusp search
- Bursts associated with Vela pulsar glitch, or high-energy neutrinos

“Excess Power” Search Methods

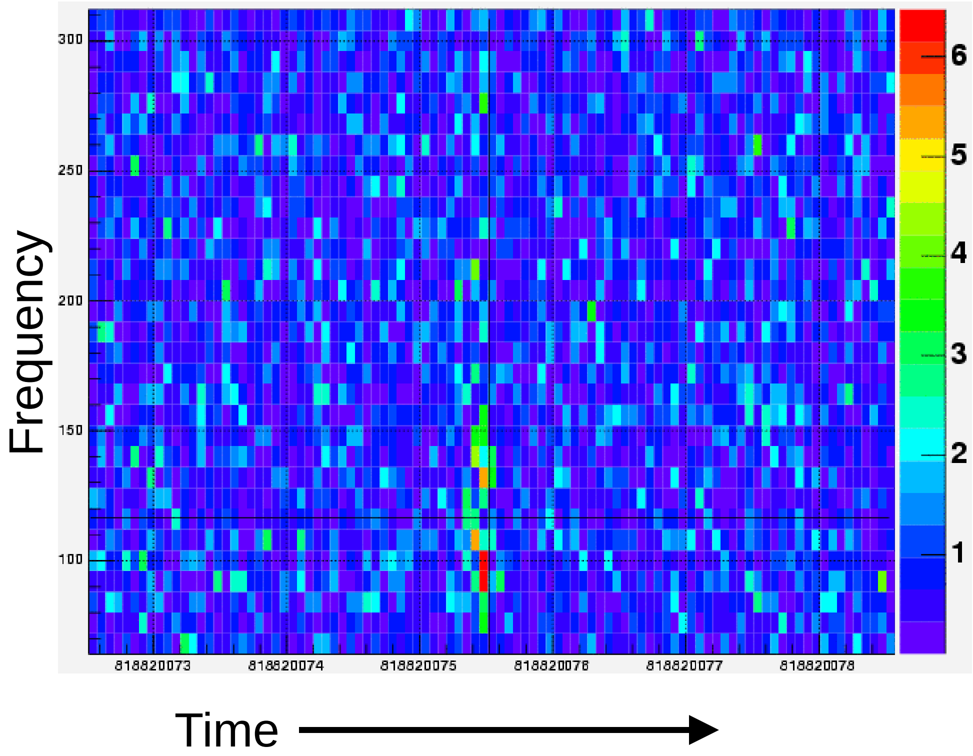
Decompose data stream into time-frequency pixels

Fourier components, wavelets, “Q transform”, etc.

Several implementations of this type of search

Normalize relative to noise as a function of frequency

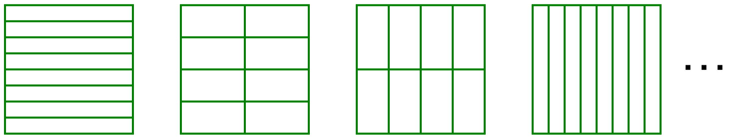
Look for “hot” pixels or clusters of pixels



Example: “WaveBurst” Method As Used for S4 All-Sky Burst Search

WaveBurst processed all 3 GW data channels simultaneously

- When all 3 interferometers in “science mode”
- Wavelet decomposition from 64–2048 Hz with 6 different resolutions from 1/16 sec × 8 Hz to 1/512 sec × 256 Hz



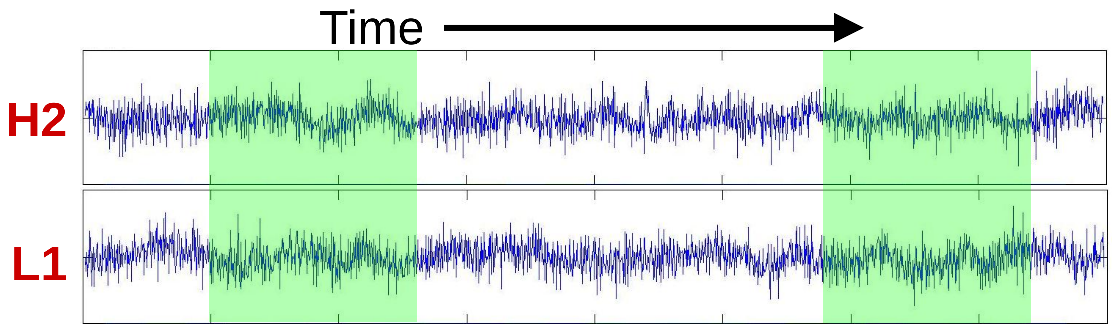
- Pixel power thresholding → select “black pixels”
- Cross-stream pixel coincidence
- Clustering of coincident pixels to build up the event
- Signal parameter estimation: time, duration, frequency, amplitude

Found coincident clusters for true time series plus 98 time shifts

- Hanford-Livingston relative times shifted by multiples of 3.125 sec
- Used to study / estimate background

Cross-Correlation Method: "CorrPower"

Look for same signal buried in two data streams by calculating the correlation between them



Checks for consistent shape, regardless of relative amplitude

Integrate over a time interval comparable to the target signal (try a few)

Allow for a time offset between the data streams

Technical note: notch out sharp spectral features such as violin modes, which can give spurious cross-correlation

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Coherent Burst Analysis

Each detector measures a linear combination of $h_+(t)$ & $h_\times(t)$ *
with antenna response factors and relative time delay depending on direction of arrival

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} F_1^+ & F_1^\times \\ F_2^+ & F_2^\times \\ \vdots & \vdots \\ F_N^+ & F_N^\times \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix}$$

data = response × signal + noise

$\Rightarrow$ Data from 2 sites can uniquely determine $h_+(t)$ and $h_\times(t)$ for an arbitrary signal, *in the absence of noise and if the arrival direction is known*

$\Rightarrow$ Data from 3 or more sites *over-determines* $h_+(t)$ and $h_\times(t)$ if the arrival direction is known

** Assuming that GR is correct !*

Geometric View of Coherent Analysis

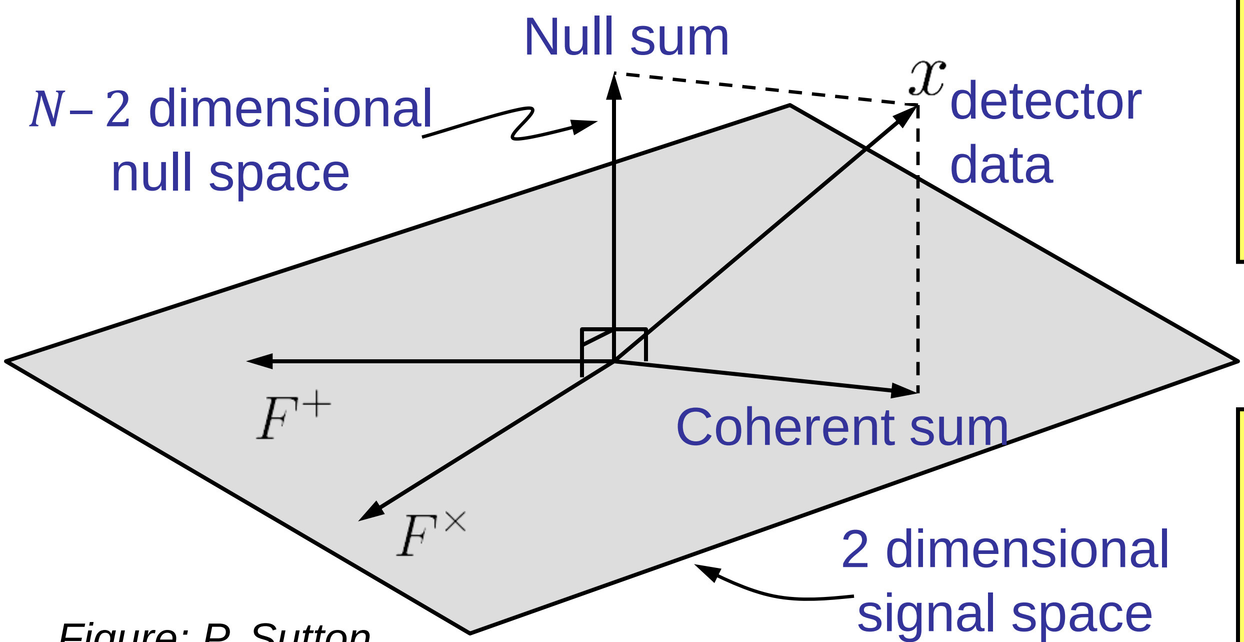


Figure: P. Sutton

Coherent sum:
Find linear combination of detector data that maximizes signal to noise ratio

Null sum:
Linear combination of detector data that has no GW signal—provides consistency test

Treat this as a maximum likelihood problem

Consider all possible sky positions (arrival directions)

Find the sky position, $h_+(t)$ & $h_\times(t)$ with the greatest likelihood for producing the data that was recorded

Pitfall of Searching the Whole Sky

Issue: For some sky positions, network effectively sees only one polarization component

- The other polarization component is unconstrained or poorly constrained
- Usually where one of the detectors has null response

Solution: use a regularization scheme

- Effectively penalizes “un-physical” solutions
- “Hard constraint”: only look at the well-determined polarization component
- “Soft constraint”: de-weight the poorly-determined polarization component
- etc.
- Best choice depends in part on the nature of the instrumental glitches

All-Sky Generic GW Burst Search

Analyzed all LIGO and Virgo collected since 2005 when at least two detectors were running

- Total live observation time: 636 days
- LIGO+Virgo coherent analysis
- GEO data often available for investigating possible event candidates

Sensitive to arbitrary GW signals in the range 64–5000 Hz

- Background measured by analyzing data with artificial time shifts
- Event selection thresholds tuned for low false alarm probability

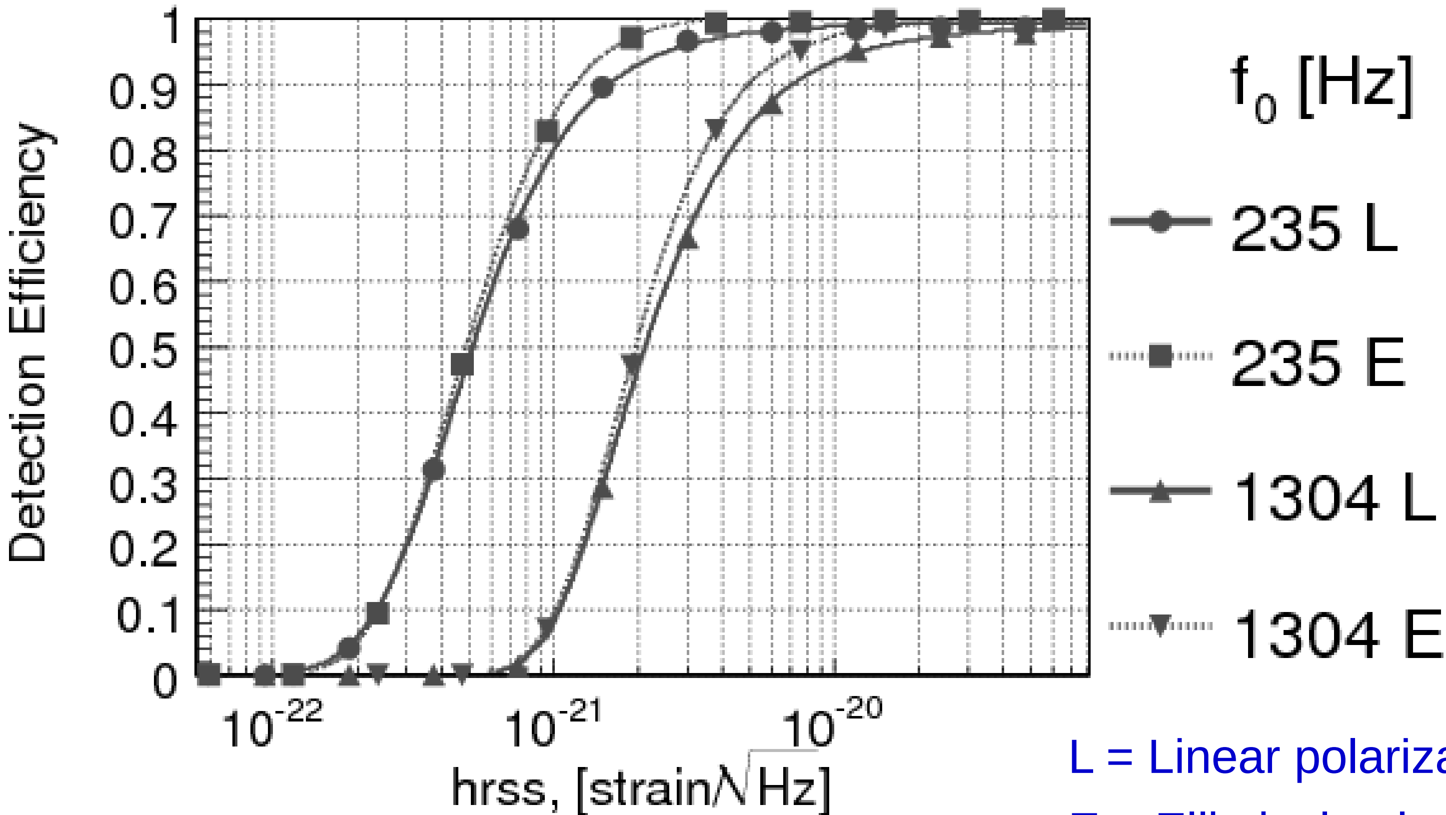
No event survived all selection cuts

We set upper limits on burst rate vs. amplitude for representative waveforms using Monte Carlo

Abadie et al., Phys. Rev. D in press, arXiv:1202.2788

Sample Detection Efficiency Curves

For simulated signals with random times and sky positions added to real detector noise

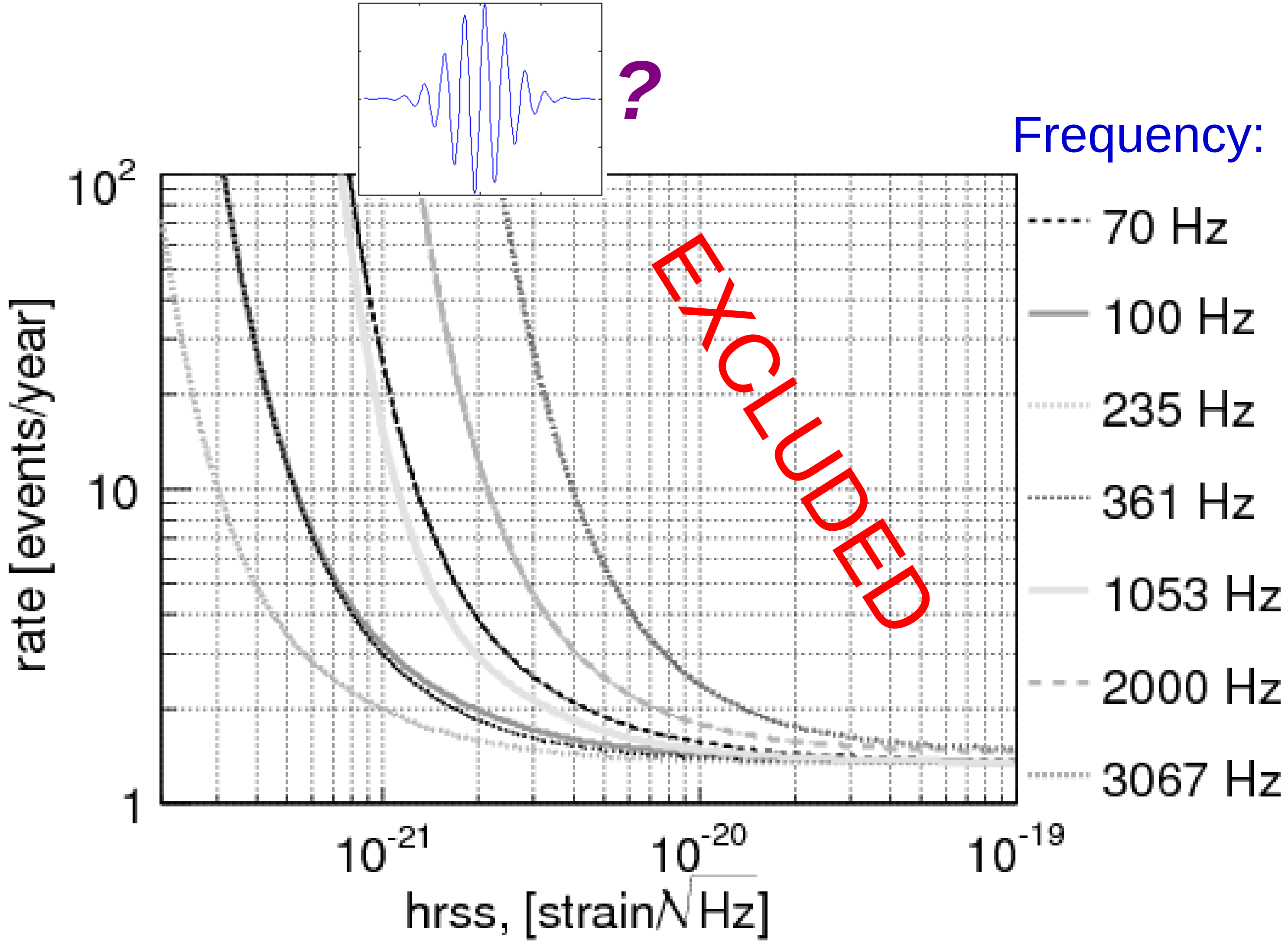


(GW burst amplitude measure)

L = Linear polarization at Earth

E = Elliptical polarization from random inclination of axis of presumed rotating source

Rate Limit vs. Amplitude



How Sensitive are Burst Searches?

Not as sensitive as matched filtering for a known waveform

But not *too* much worse, when the signal duration is short

Typically about a factor of 2

Can relate signal amplitude to energy emission in a general way:

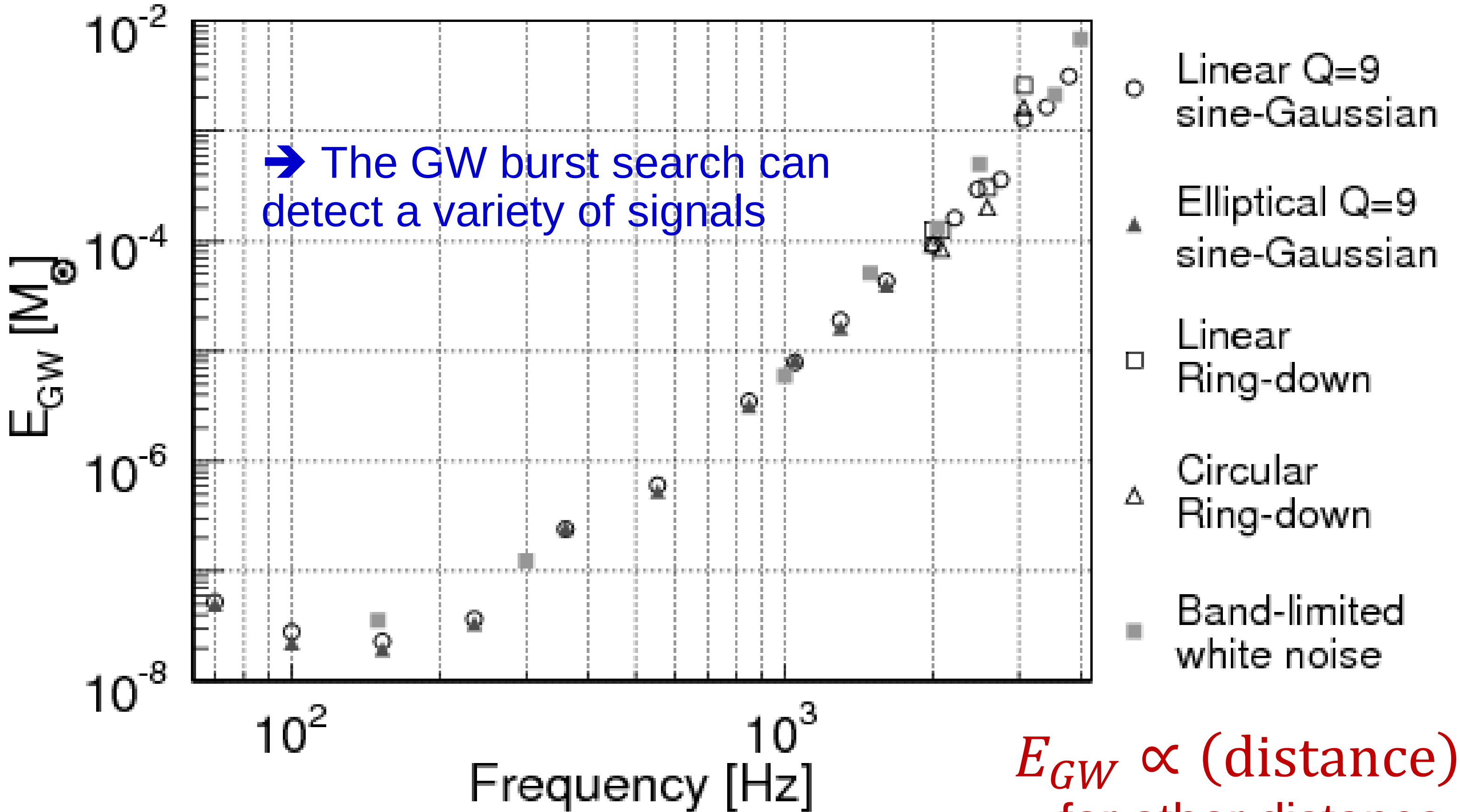
$$E_{\mathrm{GW}} = \frac{r^2 c^3}{4G} (2\pi f_0)^2 h_{\mathrm{rss}}^2$$

This assumes isotropic emission – unphysical, but fine for a rough estimate

Search Sensitivity in Energy Units

GW energy emission assuming a Galactic source (10 kpc) that could have been detected with 50% efficiency

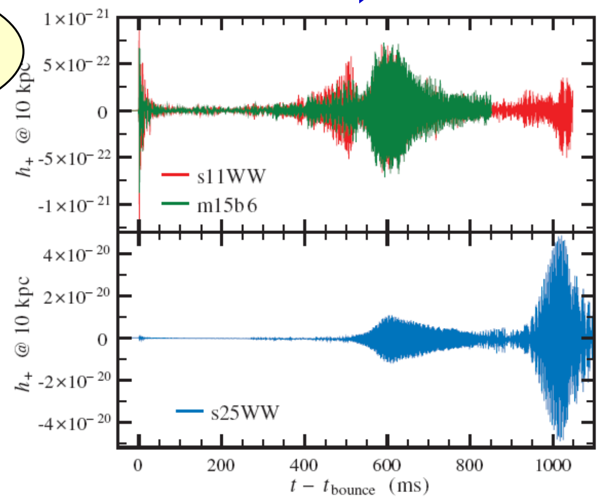
3-detector LIGO+Virgo network data, S6/VSR2+3 run



$E_{GW} \propto (\text{distance})^2$ for other distance

Order-of-Magnitude S5 Range Estimates for Supernovae and BH Mergers

Model dependent!



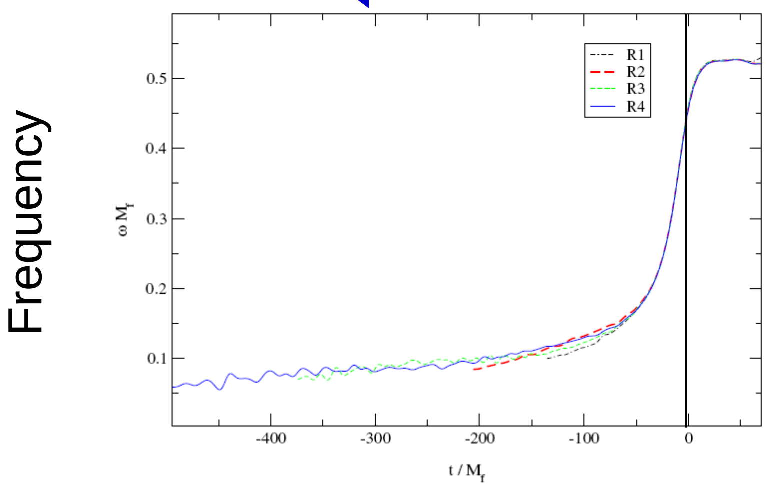


TABLE I. MODEL SUMMARY.

Model	Δt[a] (ms)	$\lvert h_{+,\max}\rvert$[b] (10^{-21})	$h_{\mathrm{char,max}}$[b,c] (10^{-21})	$f(h_{\mathrm{char,max}})$ (Hz)	E_{GW}[d] $(10^{-7} M_\odot c^2)$
s11WW	1045	1.3	22.8	654	0.16
s25WW	1110	50.0	2514.3	937	824.28
m15b6	927.2	1.2	19.3	660	0.14

$$f_{\mathrm{peak}} \approx \frac{0.46}{2\pi M_f} \approx \frac{15\ \mathrm{kHz}}{(M_f/M_\odot)}$$

11 $M_\odot$ progenitor (s11WW model)
$\Rightarrow$ reach ≈ 0.8 kpc
25 $M_\odot$ progenitor (s25WW model)
$\Rightarrow$ reach ≈ 30 kpc

Assuming ~3.5% mass radiates in the merger:
10+10 $M_\odot$ binary $\Rightarrow$ reach ≈ 6 Mpc
50+50 $M_\odot$ binary $\Rightarrow$ reach ≈ 200 Mpc

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Properties of a Stochastic GW Signal

Random signal from sum of unresolved sources

From the early universe, or from astrophysical sources since then

Usual assumptions about the signal:

Stationary

Gaussian

Unpolarized

Power-law frequency dependence, probably (e.g. f^{-3})

May be isotropic, or not

Looks basically like extra noise in each detector !

How to Search for a Stochastic Signal

Use cross-correlation between GW data streams

- No time delay for all-sky isotropic search – will affect correlation
- For anisotropic ("radiometer") search, fix time delay between streams

Include a filter function in the cross-correlation

$$Y := \int_{-T/2}^{T/2} dt_1 \int_{-T/2}^{T/2} dt_2 \, x_1(t_1) x_2(t_2) K(t_1 - t_2)$$

↓

$$Y = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \, \delta_T(f - f') \widetilde{x}_1^*(f) \widetilde{x}_2(f') \widetilde{K}(f')$$

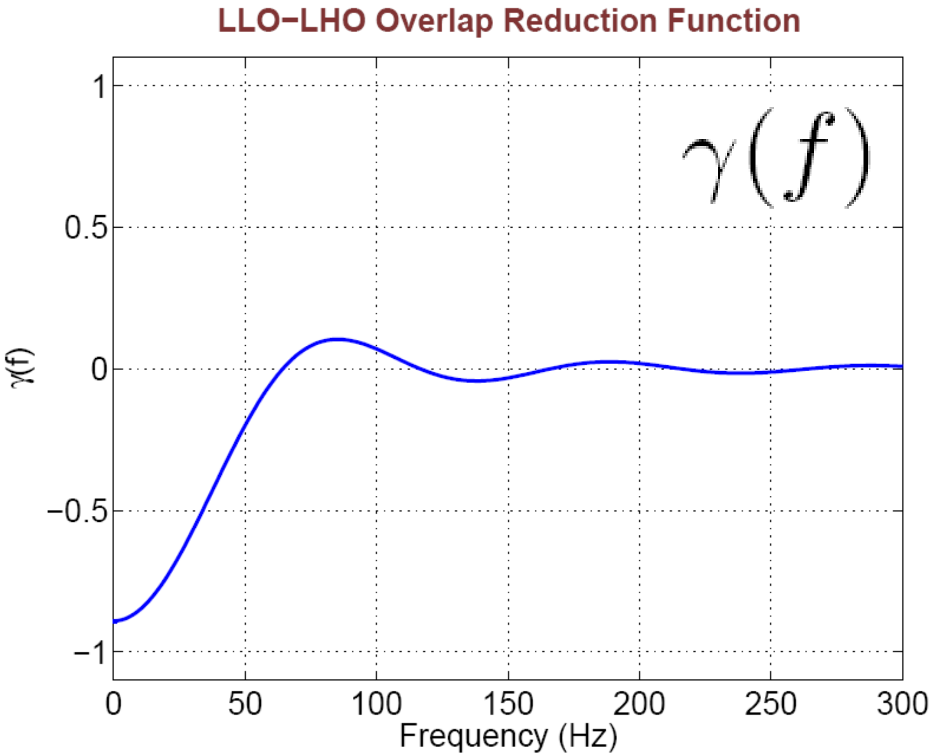
Filter function optimizes the detection statistic, accounting for two effects:

- Power spectrum of the signal being searched for
- Expected correlation between detectors, which depends on frequency due to their separation

Overlap Reduction Function
for isotropic search

Calculate expected correlation as a function of frequency

e.g. for the two LIGO observatories:



Note: H1 and H2 should have perfect correlation and thus make the most sensitive measurement, but have to worry about instrumental noise correlations

Optimal Filter for Stochastic Search

Choose:

Power spectrum of the GW signal

$$\widetilde{K}(f) = \frac{\gamma(f) P_{\rm gw}(f)}{P_{n_1}(f) P_{n_2}(f)}$$

Noise power spectra

Then signal-to-noise ratio is:

$$\left(\frac{S}{N}\right)_{\rm opt} = \sqrt{T} \left[\int_{-\infty}^{\infty} df \, \frac{\gamma^2(f) P_{\rm gw}(f)}{P_{n_1}(f) P_{n_2}(f)}\right]^{1/2}$$

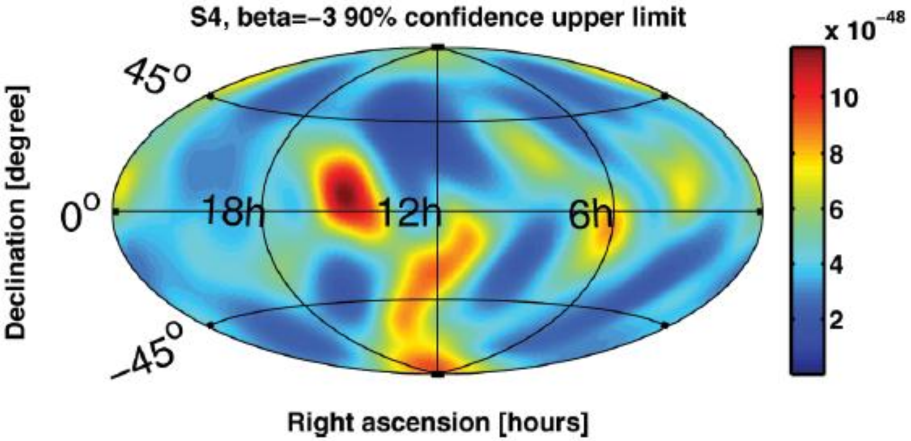
Interpret isotropic search in terms of the energy density of gravitational waves, relative to the critical energy density needed to close the universe:

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c} \frac{{\rm d}\rho_{\rm GW}}{{\rm d}f} \quad \Longrightarrow \quad P_{\rm gw}(f) = \frac{3H_0^2}{20\pi^2} \frac{\Omega_{\rm gw}(f)}{f^3}$$

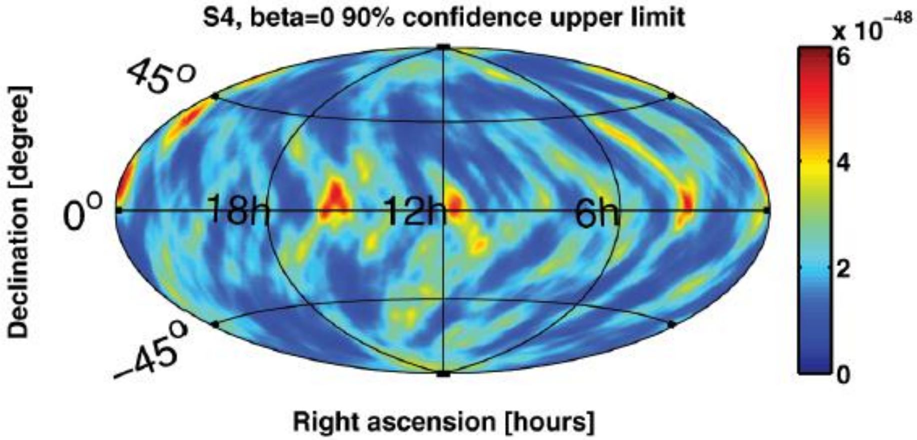
Directional Stochastic Searches

Cross-correlation with different filtering

"Radiometer" search, optimized for finding point sources:



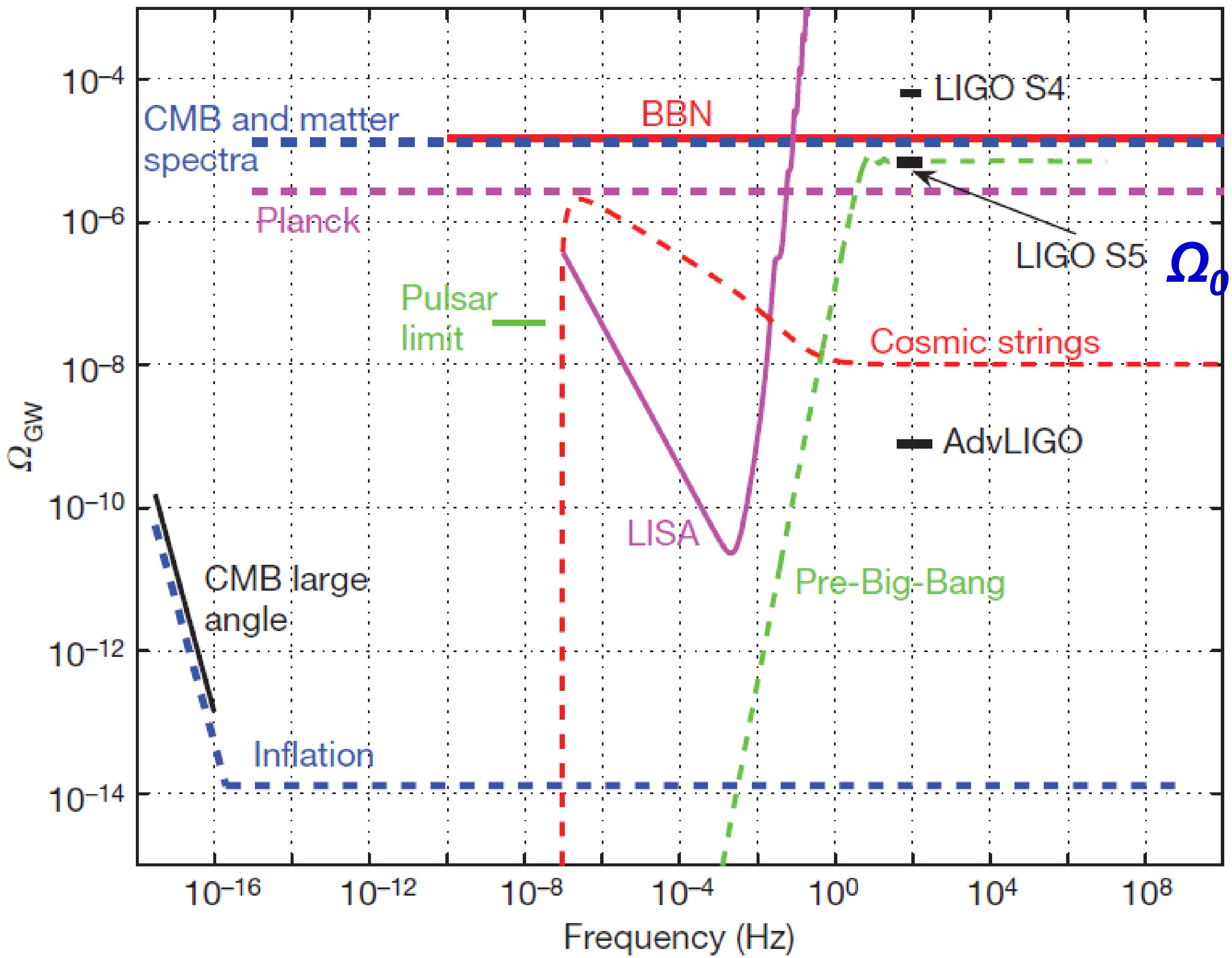
Assuming constant GW energy spectrum



Assuming constant GW strain spectrum

Another kind of directional search: spherical harmonic decomposition of the sky

Isotropic Stochastic Search Results



LSC+Virgo, Nature **460**, 990 (2009)

$\Omega_0 < 6.9\times10^{-6}$ assuming flat GW energy spectrum

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The Gravitational Wave Signal Tableau

Short duration ⟷ Long duration

Waveform known ↕ Waveform unknown

Cosmic string cusp / kink

NS / BH ringdown

Low-mass inspiral

Asymmetric spinning NS

LISA binary

High-mass inspiral

Rotation-driven instability

Binary merger

Cosmological stochastic background

Stellar core collapse

Astrophysical stochastic background

???

???

???

Summary of Data Analysis Methods