Gravitational-Wave Data Analysis: Lecture 2

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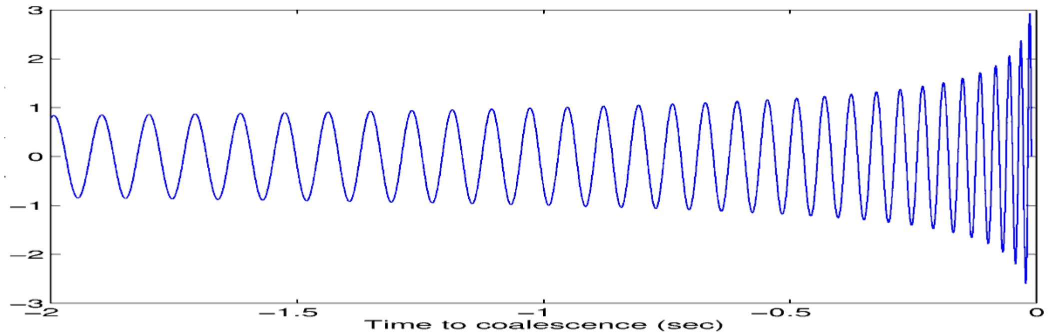
Outline for Today

- Matched filtering in the time domain
- Matched filtering in the frequency domain
- Optimal noise weighting
- Filter banks for inspiral signals
- Consistency tests
- Searches for continuous-wave signals

Searching for Known Waveforms

Example: low-mass inspiral

h(t)



$$h(t) = A(t)\cos(\,\Psi(t)\,)$$

Waveform known well, or fairly well, in some parametrized space

e.g. inspiral with 1.4+1.4 $M_\odot$

or with 10+1.4 $M_\odot$

Another example: continuous-wave emission from a spinning neutron star

Phase Evolution of an Inspiral

Accurate knowledge of the phase is crucial for matched filtering

Orbital phase vs. time → orbital phase vs. frequency during chirp

"Post-Newtonian expansion" if spins are negligible:

$$\Psi(f) = 2\pi f t_c + \frac{3}{128\eta}(\pi m f)^{-5/3}$$ Newtonian

$$+ \frac{5}{96\eta}\left(\frac{743}{336} + \frac{11}{4}\eta\right)(\pi m f)^{-1}$$ 1PN

$$- \frac{3\pi}{8\eta}(\pi m f)^{-2/3}$$ 1.5PN

$$+ \frac{15}{64\eta}\left(\frac{3058673}{1016064} + \frac{5429}{1008}\eta + \frac{617}{144}\eta^2\right)(\pi m f)^{-1/3}$$ 2PN

$$+ \cdots$$

Relativistic effects

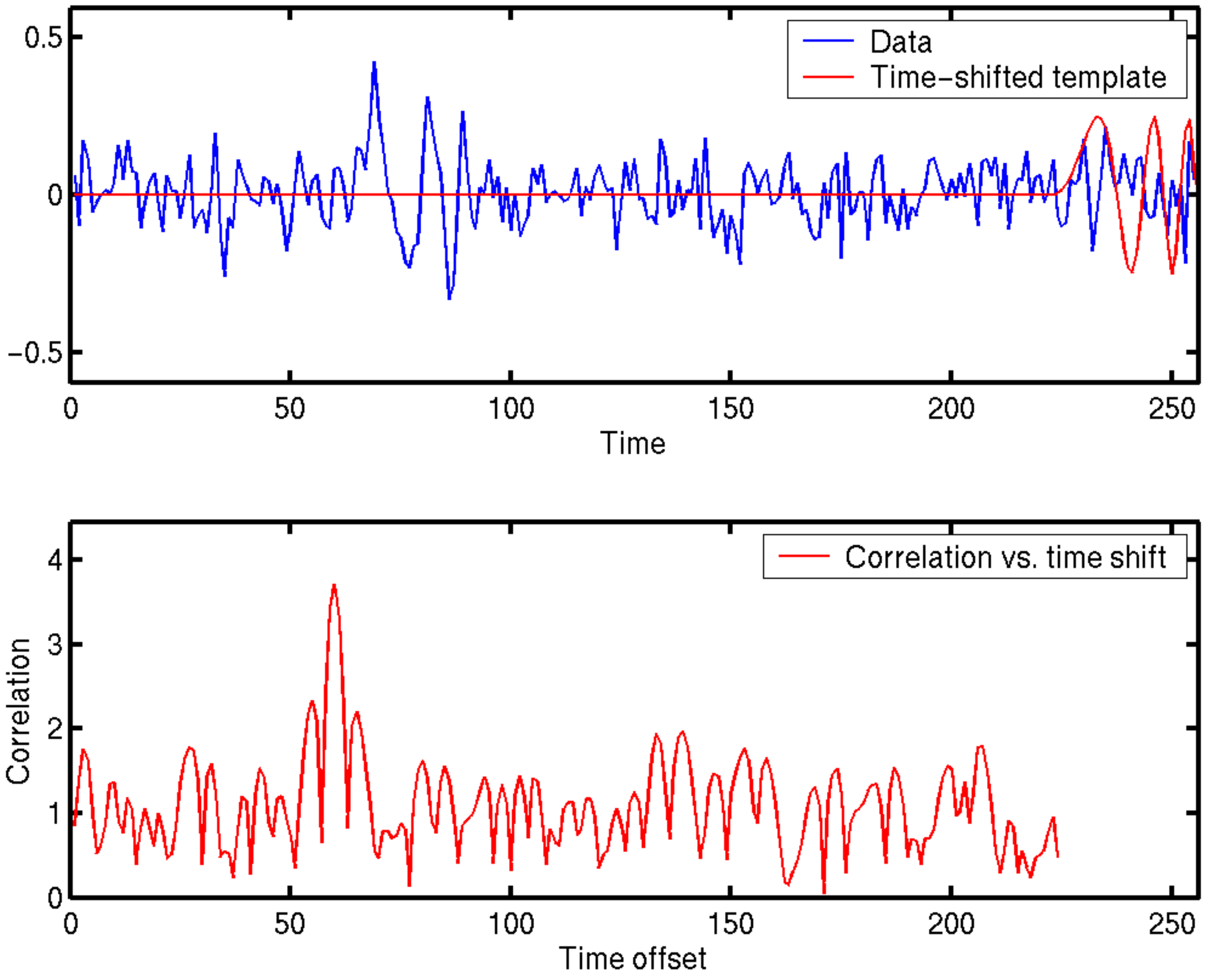
where $m = (m_1 + m_2)\,,\quad \eta = \dfrac{m_1\, m_2}{m^2}$ **and "chirp mass" is** $m\,\eta^{3/5}$

Inspiral Phase to 3.5PN

$$
\begin{aligned}
\Psi(f;M,\eta) \;=\;& 2\pi f t_C - 2\phi_C - \pi/4 \\
&+ \frac{3}{128\eta\, v^5}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right) v^2 - 16\pi v^3 + \left(\frac{15\,293\,365}{508\,032} + \frac{27\,145}{504}\eta + \frac{3085}{72}\eta^2\right) v^4\right. \\
&+ \pi\left[\frac{38\,645}{756} - \frac{65}{9}\eta\right]\left[1 + 3\ln\left(\frac{v}{v_0}\right)\right] + \left\{\frac{11\,583\,231\,236\,531}{4\,694\,215\,680} - \frac{640}{3}\pi^2 - \frac{6\,848}{21}\left(\gamma + \ln(4\,v)\right)\right. \\
&+ \left.\left(-\frac{15\,335\,597\,827}{3\,048\,192} + \frac{2\,255}{12}\pi^2\right)\eta + \frac{76\,055}{1\,728}\eta^2 - \frac{127\,825}{1\,296}\eta^3\right\} v^6 \\
&+ \left.\pi\left[\frac{77\,096\,675}{254\,016} + \frac{378\,515}{1\,512}\eta - \frac{74\,045}{756}\eta^2\right] v^7\right\},
\end{aligned}
$$

… where $\upsilon = (\pi M f)^{1/3}$

Basic Illustration of Matched Filtering



General Comments About Matched Filtering

Correlating data with template is equivalent to an FIR filter with coefficients following the template

The impulse response of that FIR filter looks like the template, but *time-reversed*

The goal of this kind of filter is to “compress” an extended signal into a delta function

Phase coherence is more important than amplitude matching

Also known as “Wiener optimal filter”

Optimal detection statistic if noise is Gaussian

Source Parameters vs. Signal Parameters

Inspiral source parameters

Masses (m1, m2)

Spins → Negligible for neutron stars, at least

Orbital phase at coalescence → Maximize analytically when filtering

Inclination of orbital plane

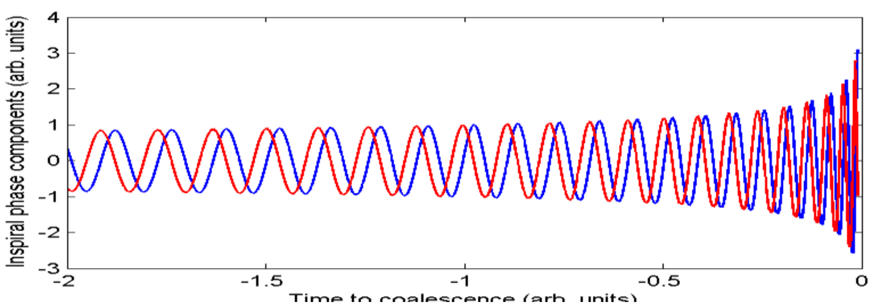
Sky location

→ Simply multiplicative for a given detector (long-wavelength limit)

Distance → Simply multiplicative

Coalescence time

Filter with orthogonal templates, take quadrature sum



➔ Only have to explicitly search over masses and coalescence time (“intrinsic parameters”)

Matched Filtering in Frequency Domain

$$C(t) = \int_{-\infty}^{\infty} dt'\, s(t')\, h(t'-t)$$

Time offset

Data

Template with time offset

Rewrite correlation integral using Fourier transforms…

$$\Rightarrow \quad C(t) = 4\int_{0}^{\infty} \tilde{s}(f)\, \tilde{h}^{*}(f)\, e^{2\pi i f t}\, df$$

This is simply the inverse FFT of $\tilde{s}(f)\, \tilde{h}^{*}(f)$

Computationally efficient way to calculate fliter output for a range of times!

Optimal Matched Filtering with Frequency Weighting

FFT of data

Template; can be generated in frequency domain using stationary phase approximation

$$C(t) = 4 \int_0^\infty \frac{\tilde{s}(f)\ \tilde{h}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

Noise power spectral density

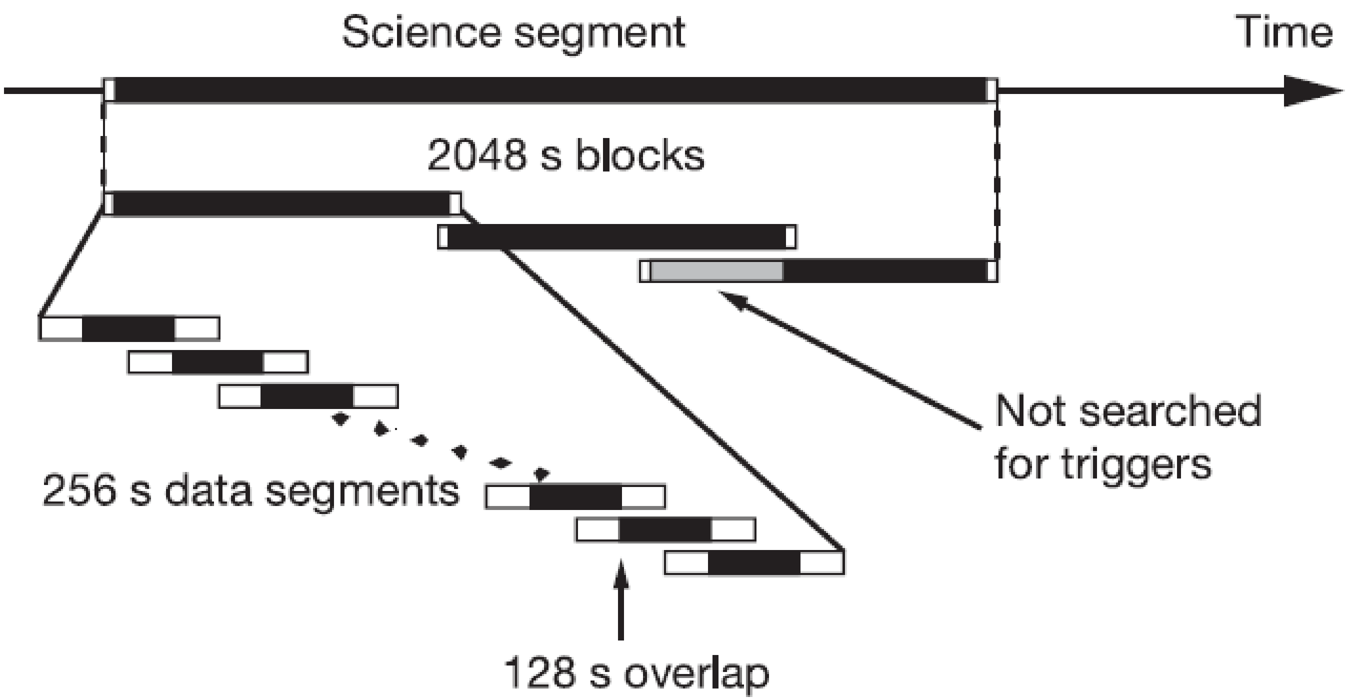
Look for maximum of $|C(t)|$ above some threshold ➔ trigger

Searching a Full Data Set

Search overlapping intervals to cover science segment, avoid wrap-around effects

Do inverse FFTs on, say, 256 s of data at a time

Estimate power spectrum from bin-by-bin median of fifteen 256-sec segments



Template Matching

Want to be able to detect any signal in a *space* of possible signals

All with different phase evolution

… but do it with a finite set of templates!

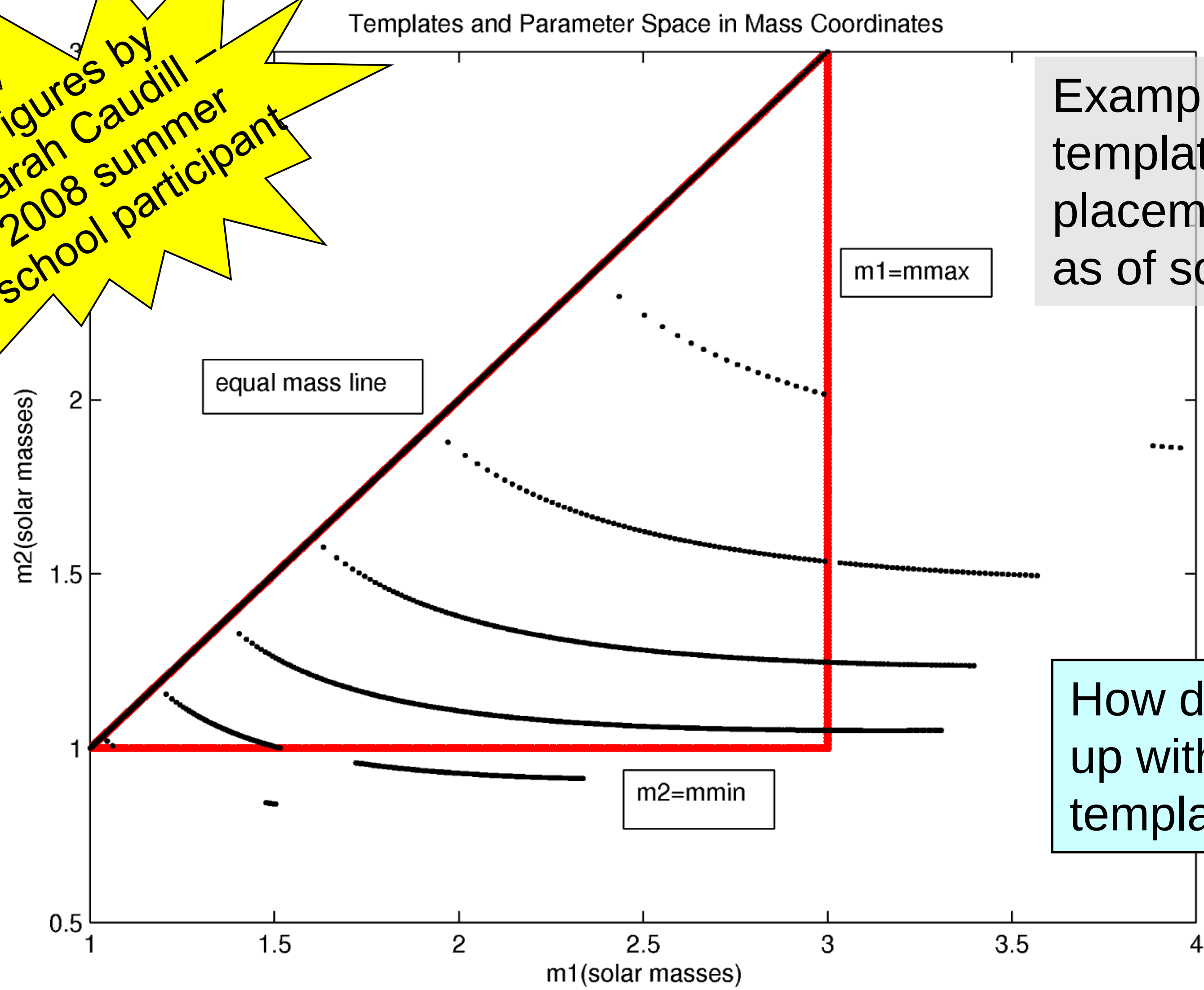
So make sure there is a "close enough" template for every part of the signal space

Require a minimum overlap between signal and template, e.g. 0.97

Often can calculate a "metric" which parametrizes the mismatch for small mismatches

Template Bank Construction

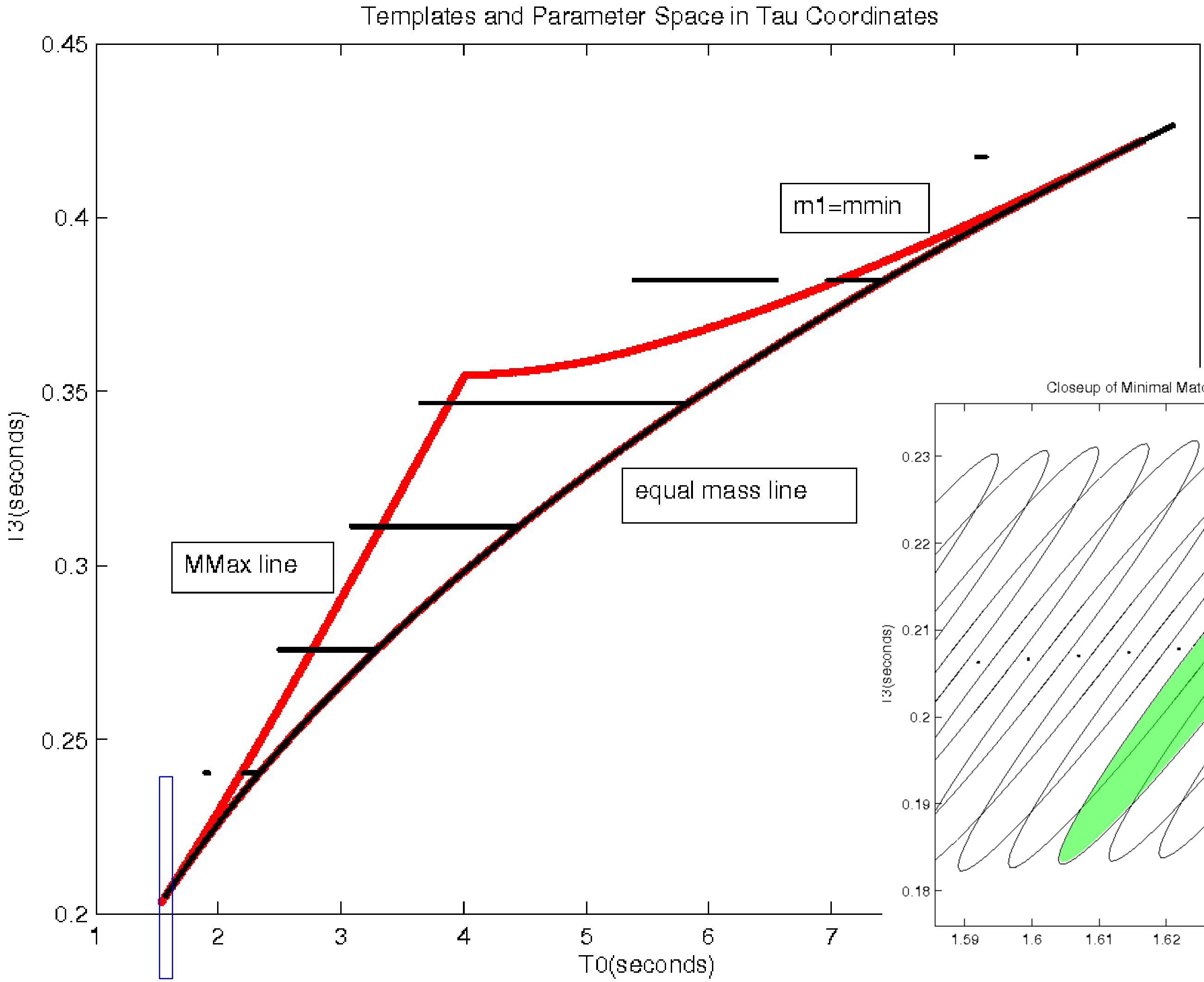
Figures by Sarah Caudill – 2008 summer school participant



Example from LAL template bank placement algorithm as of some years ago

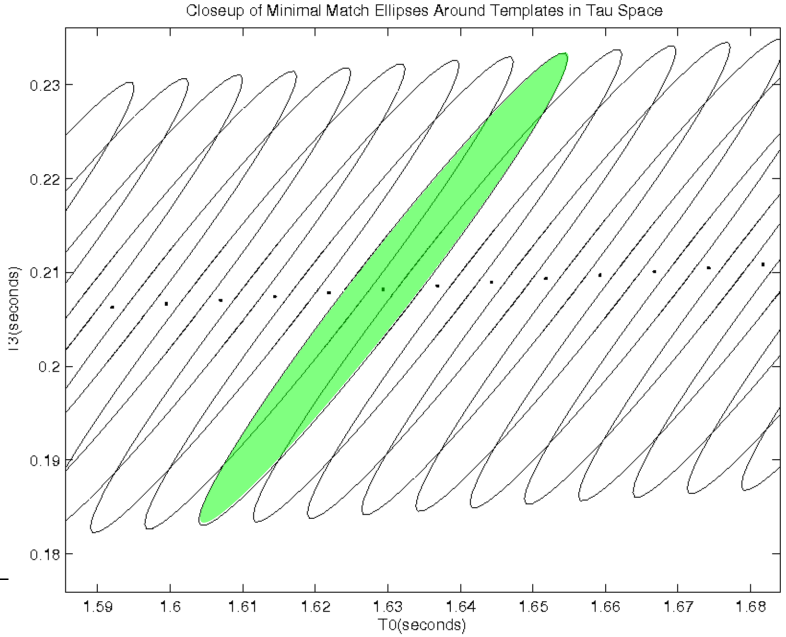
How did we come up with this set of templates???

Template Bank Construction in (τ_0,τ_3) space

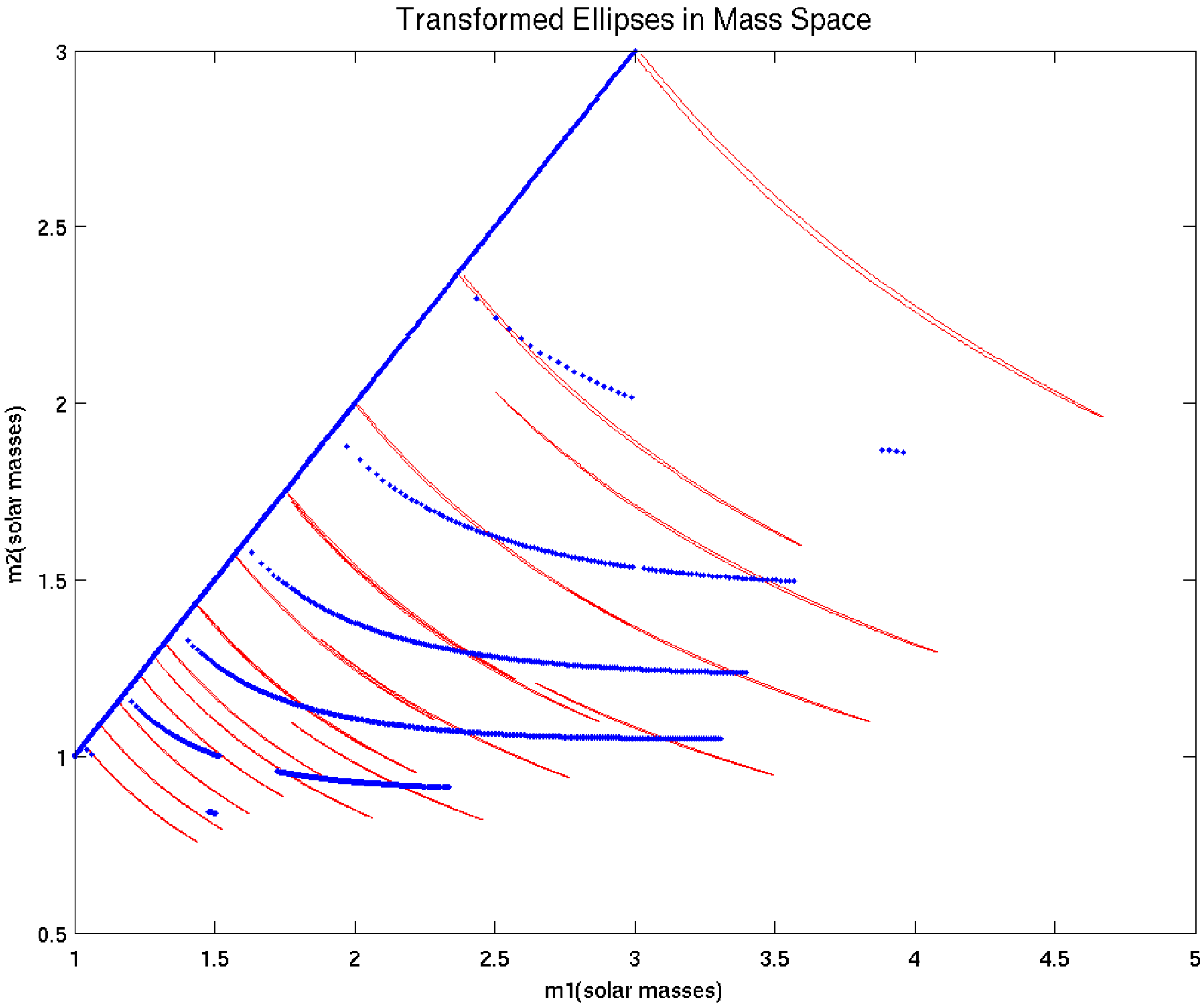


$$\tau_0 \propto m^{-5/3} / \eta$$

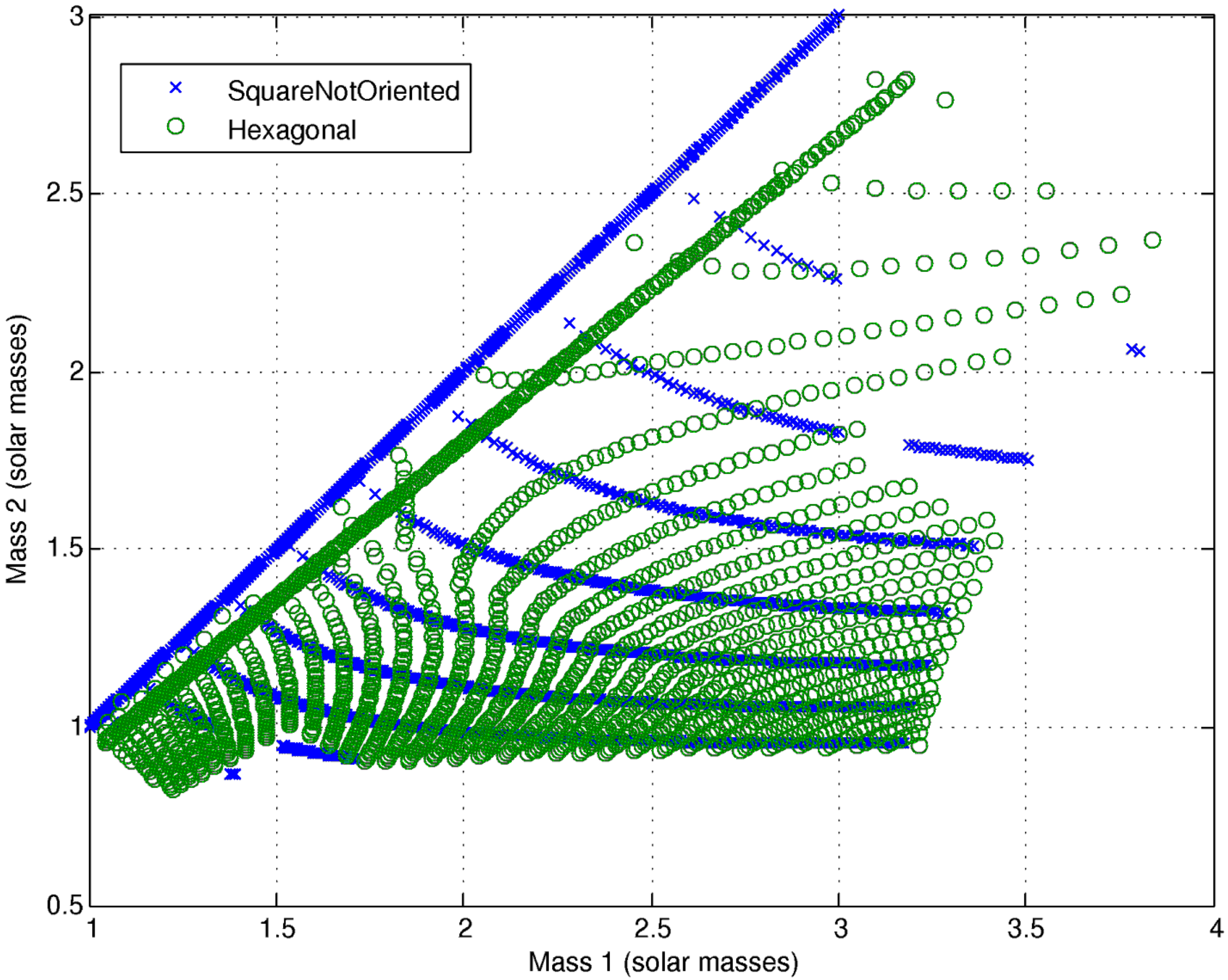
$$\tau_3 \propto m^{-2/3} / \eta$$



Ellipses in Mass Space

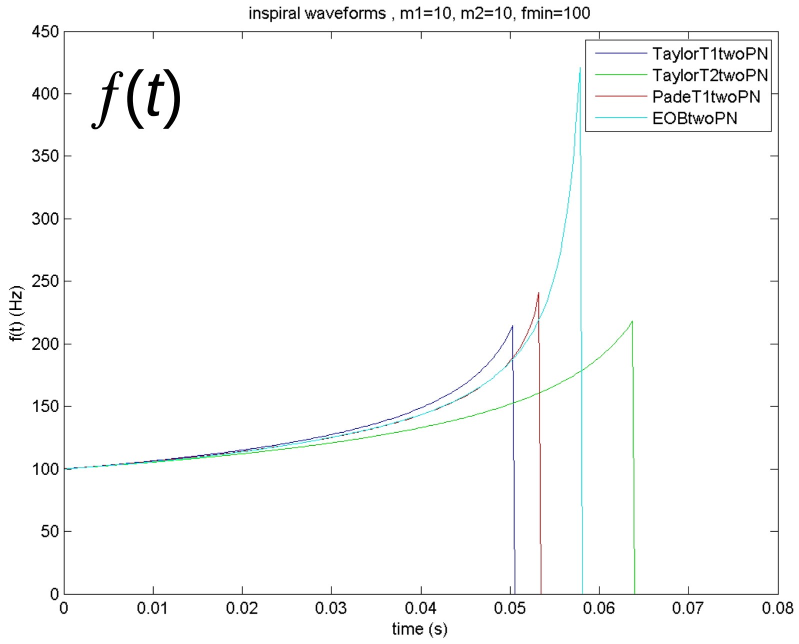


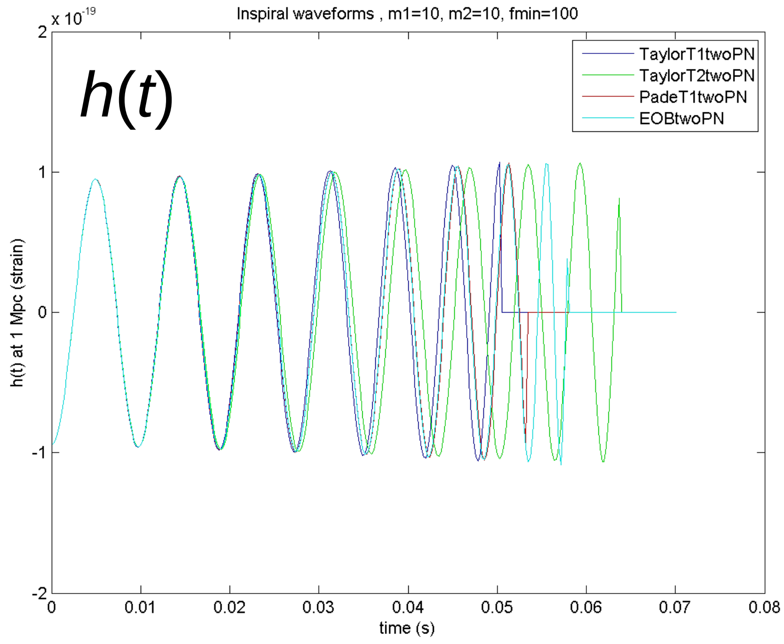
Different Bank Layout Methods



Less-certain Analytic Waveforms for High-Mass Inspirals

Different analytic approximations for 10+10 $M_\odot$ black hole binary



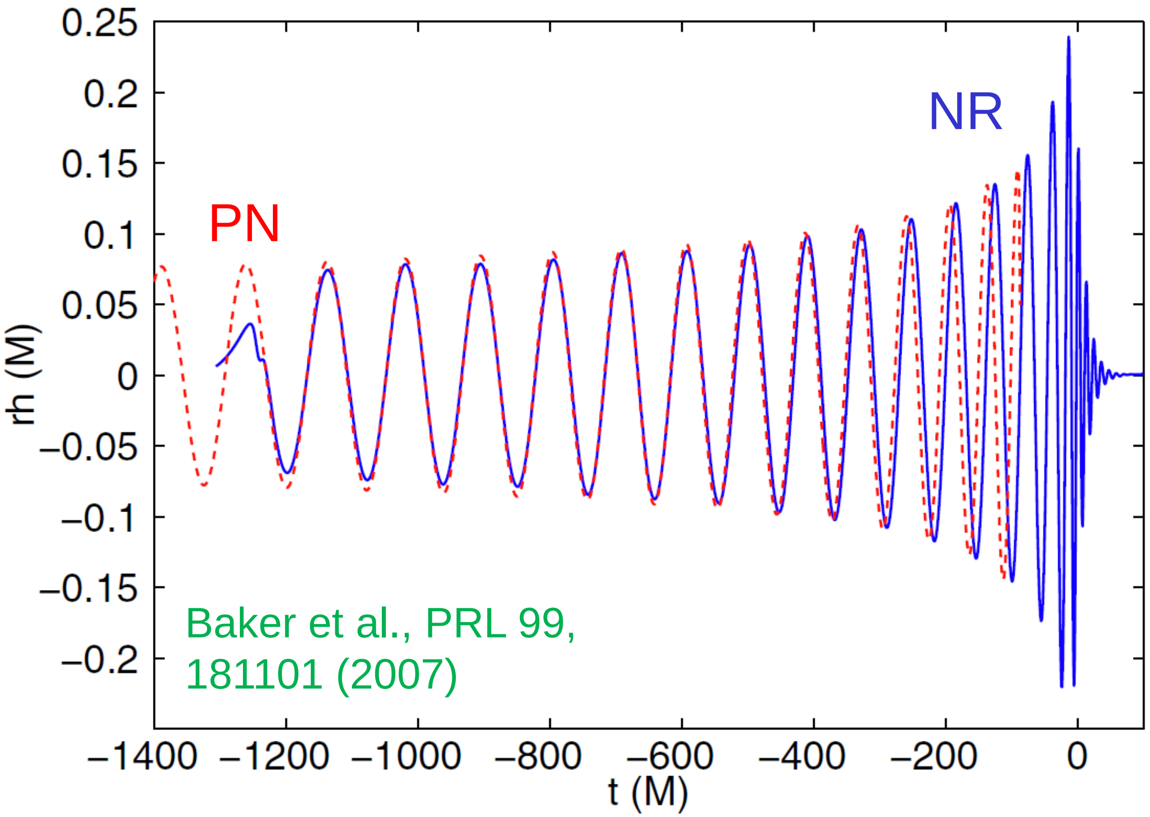


Also, black hole spin can have a large effect on the waveform

Numerical Relativity to the Rescue !

It's now possible to accurately calculate final stages of inspiral, merger, and subsequent ringdown

Can construct "hybrid" waveforms, e.g.:



Templates for Detection vs. Parameter Estimation

Could use a bank of templates in an enlarged parameter space

e.g. Buonanno, Chen, and Vallisneri, Phys. Rev. D 67, 104025 (2003)

$$h(f)=f^{-7/6}(1-\alpha f^{2/3})\theta(f_{cut}-f)\exp[i(\phi_0+2\pi t_0 f+\psi_0 f^{-5/3}+\psi_3 f^{-2/3})]$$

Analytically calculate α to maximize SNR

Parameters of the search

This can match various waveform models rather well

In practice, post-Newtonian waveforms also work pretty well for detecting a wide range of physical waveforms

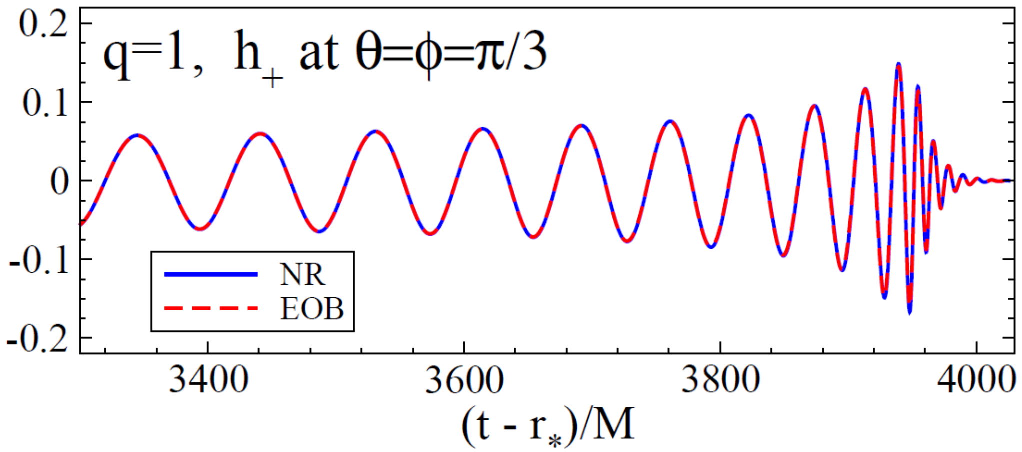
For high masses, “effective one-body” (EOB) templates work well, especially when tuned using numerical relativity calculation

Once a signal is detected, can re-filter with true physical templates to extract physical parameters

Analytic Model Tuned Using NR

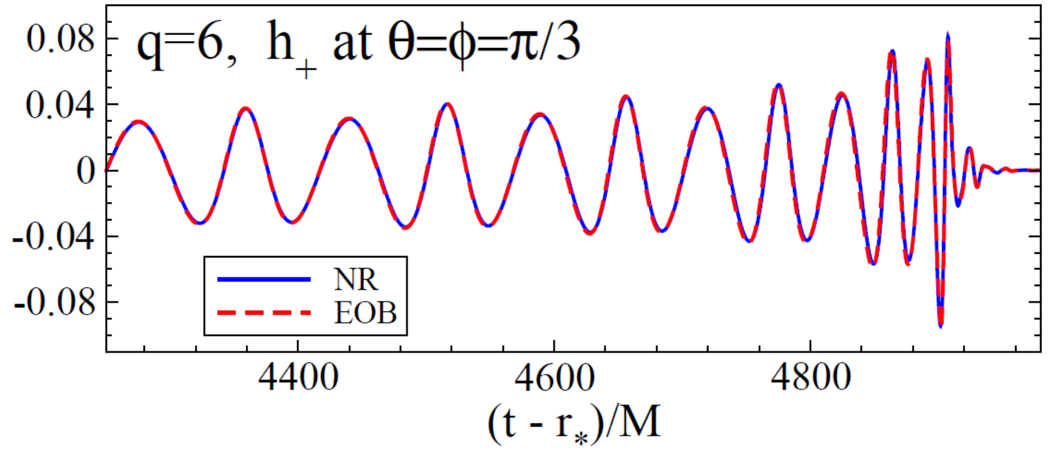
"EOBNR" : Effective One-Body model, with some parameters adjusted to match NR waveforms

Equal mass (non-spinning)

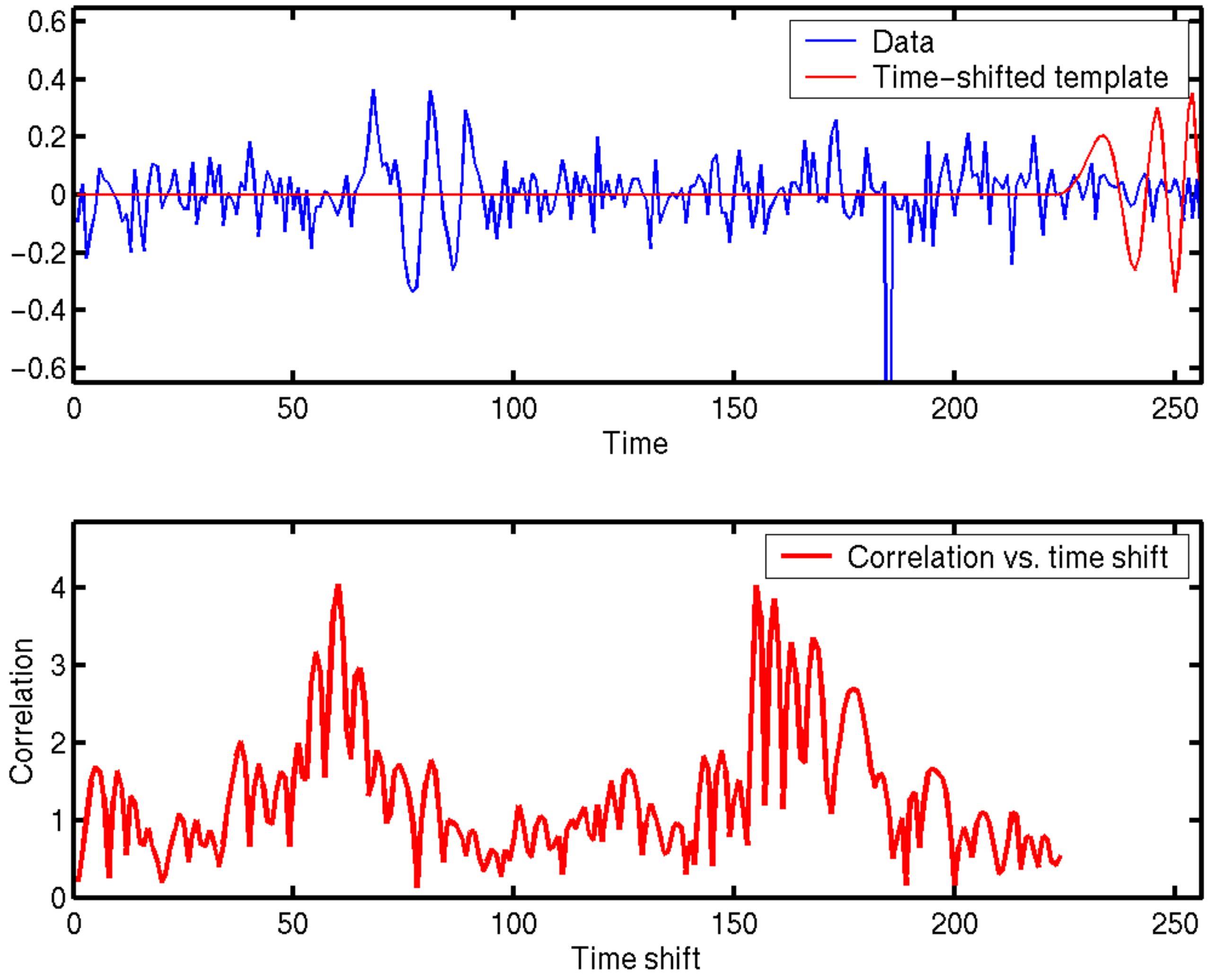


Pan et al., PRD 84, 124052 (2011)

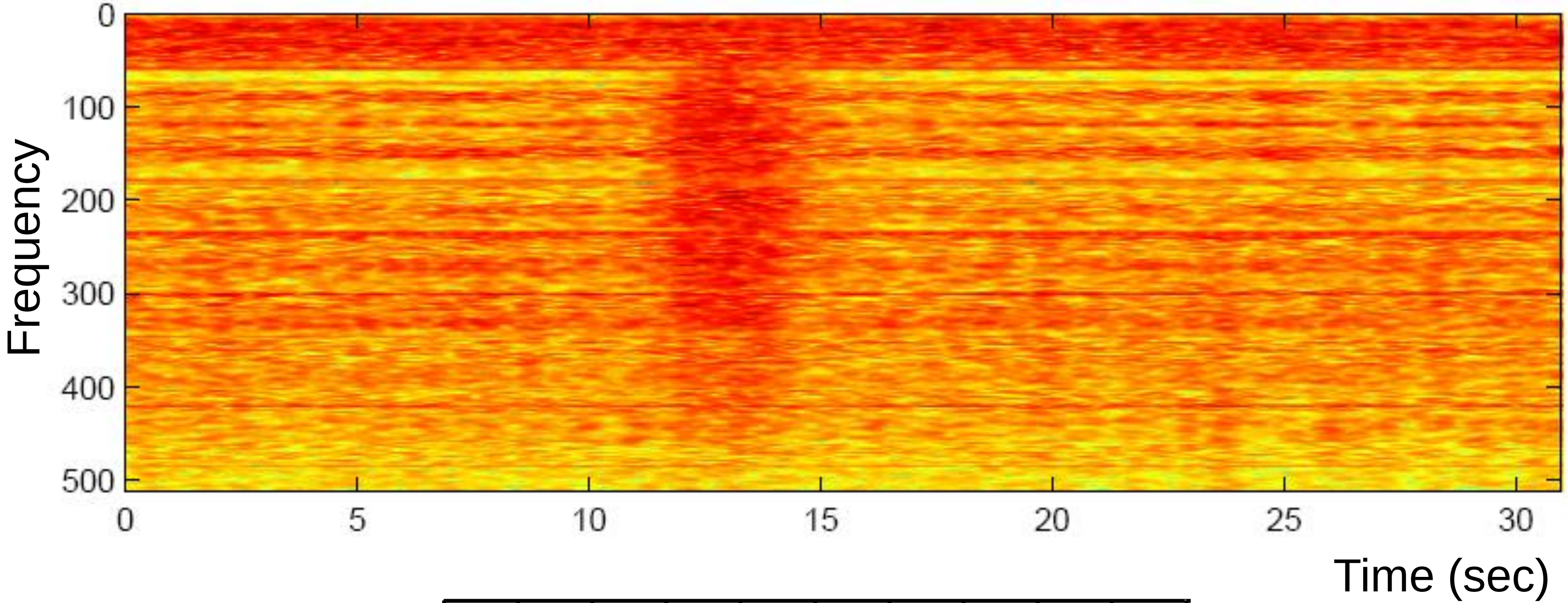
Mass ratio 6:1



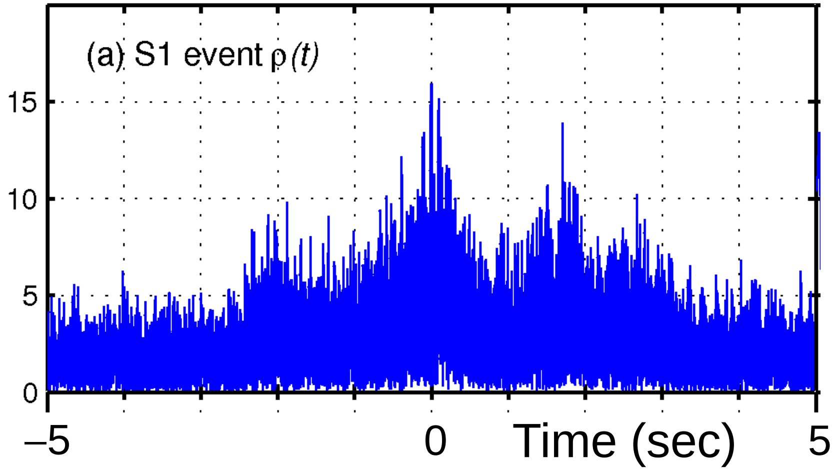
Matched Filtering
Susceptibility to Glitches



Dealing with Non-Stationary Noise



Inspiral filter output:

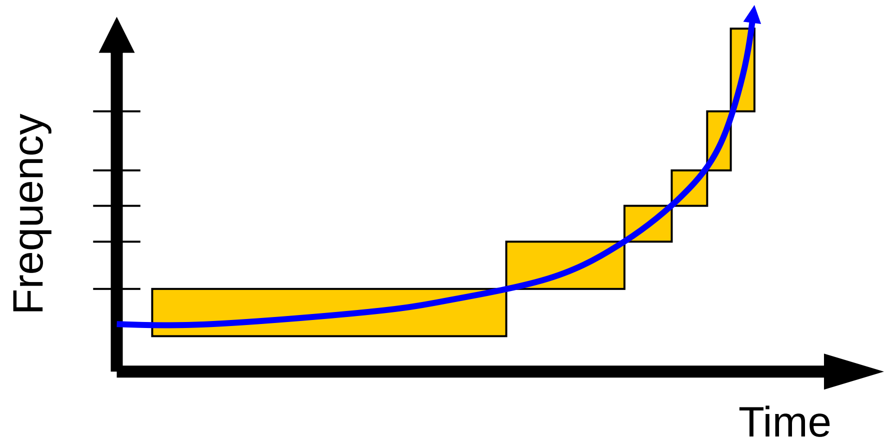


Waveform Consistency Tests

Chi-squared test

Divide template into p parts, calculate

$$\chi^2(t) = p \sum_{l=1}^{p} \| C_l(t) - C(t)/p \|^2$$



Can use χ^2 with ρ to form an "effective SNR", e.g.:

$$\rho_{\mathrm{eff}}^2 = \frac{\rho^2}{\sqrt{\left(\frac{\chi^2}{2p-2}\right)\left(1 + \frac{\rho^2}{250}\right)}}$$

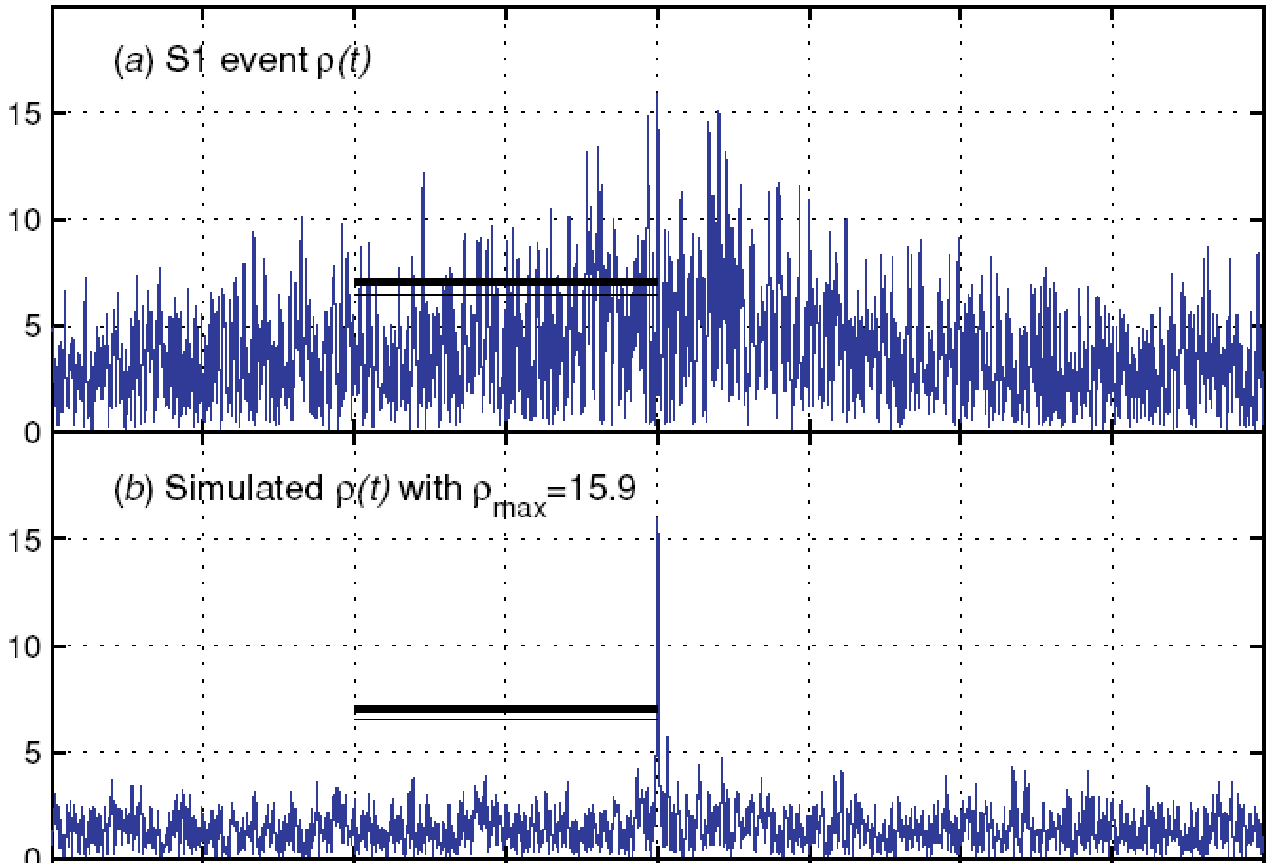
$$\rho_{\mathrm{new}} = \begin{cases} \rho, & \chi^2 \leq n_{\mathrm{dof}} \\ \dfrac{\rho}{\left[\left(1 + \frac{\chi^2}{n_{dof}}^{4/3}\right)/2\right]^{1/4}}, & \chi^2 > n_{\mathrm{dof}} \end{cases}$$

$$\hat{\rho} = \begin{cases} \dfrac{\rho}{[(1 + (\chi_r^2)^3)/2]^{1/6}} & \text{for } \chi_r^2 > 1, \\ \rho & \text{for } \chi_r^2 \leq 1. \end{cases}$$

Empirical – to separate signals from background as cleanly as possible

Tests Using Filter Output(s)

e.g. time that $\rho(t)$ or $\chi^2(t)$ spends above some threshold

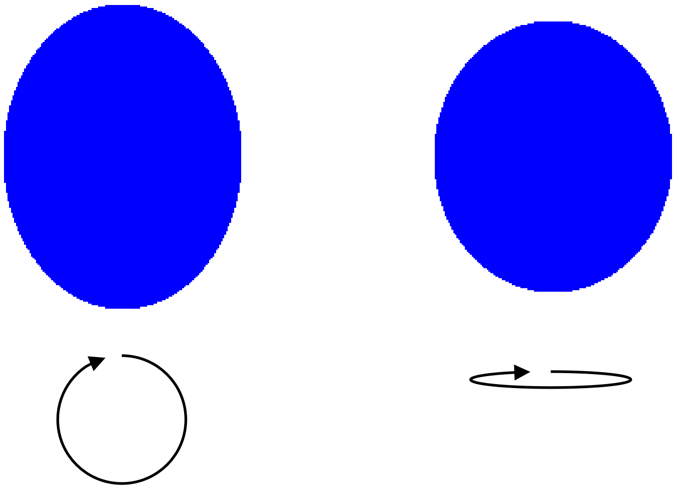


Continuous, Known Waveform: GW from Spinning Neutron Stars

If not axisymmetric, will emit gravitational waves

Example: ellipsoid with distinct transverse axes

Along spin axis: From side:



Continuous GW Signals at Earth

Start with a sinusoidal signal with spin-down term(s)

Polarization content depends on orientation/inclination of spin axis

Amplitude modulation

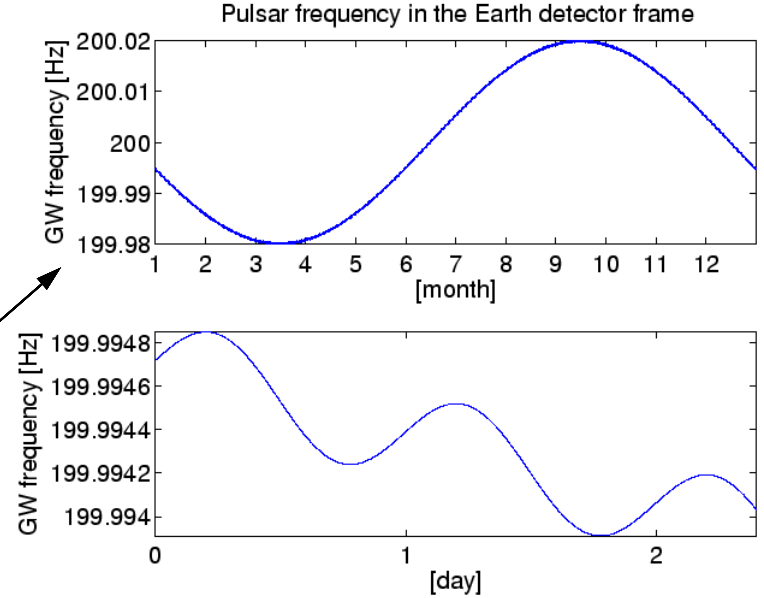
Polarization projection changes over a sidereal day

Doppler shift

$$\frac{\Delta f}{f} = \frac{\mathbf{v} \cdot \mathbf{n}}{c}$$

Annual variation: up to $\sim 10^{-4}$

Daily variation: up to $\sim 10^{-6}$



GW signals from binary systems are more complicated !

Additional Doppler shift due to orbital motion of neutron star

Varying gravitational redshift if orbit is elliptical

Shapiro time delay if GW passes near companion

Search Methods for CW signals

Several cases to consider:

Candidates

- Sky position and spin frequency known accurately — Radio pulsars
- Sky position and spin frequency known fairly well — X-ray pulsars
- Sky position known, but frequency and/or binary orbit parameters unknown — LMXBs
- Search for unknown sources in favored sky regions — Globular clusters, Galactic center, Supernova remnants
- Search for unknown sources over the whole sky — Unseen isolated neutron stars

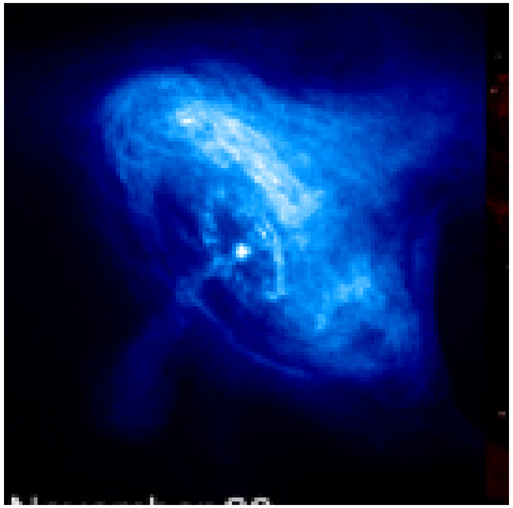
Different computational challenges $\Rightarrow$ Different approaches

Search for Gravitational Waves from Known Pulsars

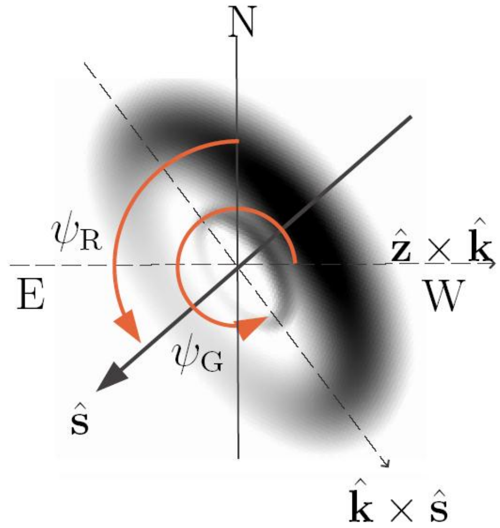
Method: heterodyne time-domain data using the known spin phase of the pulsar

- Requires precise timing data from radio or X-ray observations
- Include binary systems in search when orbits known accurately
- Exclude pulsars with significant timing uncertainties
- Special treatment for the Crab and other pulsars with glitches, timing noise

Chandra image



Model



$h_0 < 2 \times 10^{-25}$

Implies that GW emission accounts for ≤ 2% of total spin-down power

Wide Parameter Space Searches

Method: **matched filtering with a bank of templates**

Parameters:

Sky position
Spin axis inclination and azimuthal angle
Frequency, spindown, initial phase
Binary orbit parameters (if in a binary system)

Can use a detection statistic, $\mathcal{F}$, which analytically maximizes over spin axis inclination & azimuthal angle and initial phase

Even so, computing cost scales as $\sim T^{6}$

Detection threshold also must increase with number of templates

Check for signal consistency in multiple detectors

Problem: huge number of templates needed

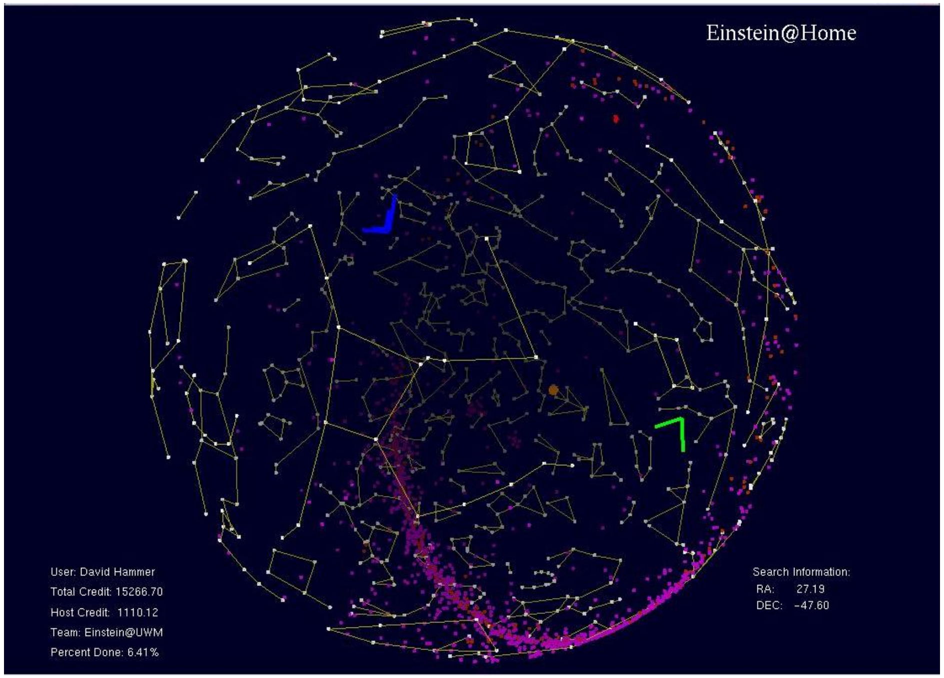
Even using clever semi-coherent analysis methods

Getting by with a Little Help from Our Friends

Public distributed computing project: Einstein@Home

einsteinathome.org

Small bits of data distributed for processing; results collected, verified, and post-processed



Searching for CW signals in LIGO+Virgo data

Also searching for millisecond pulsars in data from Arecibo, Parkes, and the Fermi satellite

So far ~320,000 users, currently providing ~500 Tflops