HW#1 —Phys776—Spring2003
Due Monday Mar. 3, 2003
Turn in at or before beginning of class.

Prof. Ted Jacobson
Room 4115, (301)405-6020
jacobson@physics.umd.edu

1. Show that conformally related metrics $g_{\mu\nu}$ and $\Omega^2(x)g_{\mu\nu}$ determine the same null geodesics, but with a different definition of affine parametrization. Show that the timelike and spacelike geodesics are *not* the same for the two metrics.

2. *Killing fields of Minkowski spacetime*

   (a) Solve Killing's equation to find three independent Killing vectors of two-dimensional Minkowski spacetime. Interpret these geometrically in terms of translations and boosts.

   (b) Show by explicit calculation that the inner product of the boost Killing field with the tangent to any affinely parametrized geodesic is constant along the geodesic.

3. *Riemann normal coordinates.* This is a special type of local inertial coordinate system at a point p in which the *second derivatives* of the metric take a particularly nice form at p. Choose an orthonormal basis of four vectors $e^\mu_{(a)}$ $(a = 0, 1, 2, 3)$ at p, and send out the collection of radial geodesics with initial tangent vector $v^a e^\mu_{(a)}$ at p. Each point x will lie at unit affine parameter on exactly one of these "radial" geodesics. Assign to x the coordinates v^a. Show that

   (a) $g_{\mu\nu}(p) = \eta_{\mu\nu}$

   (b) $g_{\mu\nu,\alpha}(p) = 0$

   (c) $g_{\mu\nu,\alpha\beta}(p) = \frac{2}{3}R_{\mu(\alpha\beta)\nu}$

   Hint: Employ the geodesic deviation equation, using the components v^a as parameters to label the family of curves in the congruence. Or maybe there is a simpler way.

4. *Synchronous or Gaussian Normal Coordinates*: For any spacetime metric, one can always find coordinates (t, x^i) such that the line element locally takes the form

$$ds^2 = -dt^2 + h_{ij}dx^i dx^j. \tag{1}$$

   $(i, j = 1, 2, 3)$ To construct such a coordinate system, start with an arbitrary 3-dimensional spacelike surface Σ_0, labeled with coordinates x^i. At each point of Σ_0 fire the geodesic orthogonal to Σ_0 and use proper time along these geodesics as the fourth coordinate. By construction on Σ_0 we have $g_{00} = -1$ and $g_{0i} = 0$ so, on Σ_0, the line element takes the above form. Show that is has this form *everywhere* (until the geodesics cross) by showing that $\partial g_{0\mu}/\partial t = 0$ as a consequence of the geodesic equation.