EXAM 1—Phy273—Fall 2001
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www.glue.umd.edu/∼tajac/273b

Prof. Ted Jacobson
Room 4117, (301)405-6020
jacobson@physics.umd.edu

2 pages, 6 problems, 50 points

Partial credit will be given for correct work shown. Also, if you miss an earlier part of a problem do not give up on later parts of the problem, even if they require the result of the earlier part. You can get partial credit by just solving the later part in a more general way.

1. Consider the complex number $z = 1 + e^{-i\pi/2}$. [2+2+2+2 = 8 *points*]
   (a) Write z in polar form.
   (b) Write z in Cartesian form.
   (c) What angle does z make with the x-axis in the complex plane?
   (d) What is the inverse of z?
2. A certain torsional oscillator has moment of inertia $I = 3\,\mathrm{kg\,m^2}$, and its potential energy as a function of the angle θ is $V(\theta) = (6\,\mathrm{J})(\theta + 2\theta^2 + \theta^3)$. [4+4 = 8 *points*]
   (a) What value of θ corresponds to a *stable* equilibrium point? (It may help to sketch $V(\theta)$.)
   (b) What is the period of small vibrations about the stable equilibrium point?
3. The figure shows the mean input power $\bar{P}$ as a function of driving frequency for a mass on a spring with damping. [2+2+2 = 6 *points*]

   (a) What is the numerical value of Q?
   (b) If the driving force is removed, the average energy decreases according to the equation $E = E_0 e^{-\gamma t}$. What is the value of γ?
   (c) If the driving force is removed, what fraction of the energy is lost per cycle? (Neglect the difference between the damped and undamped periods.)

4. A series RC circuit is driven by an emf $\mathcal{E} = \mathcal{E}_0 \cos \omega t$. [4+2+2 = 8 *points*]

   (a) What is the rms voltage across the capacitor?

   (b) By what phase angle does the current in the capacitor lag or lead the applied voltage?

   (c) If the capacitor is replaced by an inductor L, for what value of L will the power delivered to the circuit be the same?

5. A damped harmonic oscillator is characterized by the parameters $m = 1$ kg, $k = 4$ N/m, and $b = 2$ N-s/m. The particle is initially at equilibrium and is struck by a sharp blow, giving it a velocity of 2 m/s before it has time to move. [2+6 = 8 *points*]

   (a) What is the rate of change of the total mechanical energy the instant after the blow is delivered?

   (b) Where is the particle after one second has elapsed?

6. A string of length 90 cm and tension 360 N vibrates in one transverse direction with amplitude 2 mm and frequency 600 Hz in the third normal mode. The string is anchored at each end to a stationary post. [4+4+4 = 12 *points*]

   (a) Sketch the displacement of the string and label the location of the all the nodes and anti-nodes.

   (b) What is the transverse displacement of the string $y(x, t)$? Give a formula in terms of x in meters and t in seconds.

   (c) What is the maximum force exerted by the string on one of the posts?