Physics 262, Final Exam, May 20, 2002, Dr. Baden

Try to do as many problems as you can, and show your work clearly. There are a lot of problems here, more than you might have guessed, the reason for this is to try to cover as much material as possible to try to measure what you've learned. I don't expect everyone to finish every problem, so please do the problems you are more sure of first, then go back to the others. Credit will not be given for answers with no work shown. Partial credit will be given. There are 170 points total you can get on the exam.

Problem 1 (10 points). Water flows thru a fire hose of diameter **5.4cm** at a rate of **0.105 m^3/s**. The fire hose ends in a nozzle that has a diameter of **2.20 cm**. Calculate the speed that water exits the nozzle.

The conservation of flow equation is Av = constant=0.105m^3/s . The area of a hose with diameter D is given by $A = \frac{pD^2}{4}$. The continuity equation is then $Av = \frac{pD^2}{4} v$ = 0.105m3/s.

Solving for the final velocity gives $v_f = 0.105 \frac{4}{pD^2} = 276.2 m/s$

Problem 2 (10 points). An astronaut on the Moon wishes to measure the local value of *g* (as in "F=mg") on the moon (g_{moon}) by timing pulses traveling down a wire that has a large mass suspended from it. Assume that the wire has a mass of **2.20 g** and a length of **1.10 m**, and that a **7.50 kg** mass is suspended from it. A pulse requires **0.015 s** to traverse the length of the wire. Calculate g_{moon} from these data, neglecting the mass of the wire.

The velocity of waves on a string of mass *m*, length *L*, and under tension *T* is given by $v = \sqrt{\frac{T}{m/L}}$. In this problem, the tension comes from gravity due to the weight *M=1.5kg*, so

T=Mg so we have $v = \sqrt{\frac{Mg}{m/L}}$. If the wave takes 6.1s to traverse the string of length 4.1m, then the velocity is *v=1.1m/0.015s=73.3m/s*. Solving for *g* gives

$$g = \frac{v^2 m}{ML} = \frac{73.3^2 m^2/s^2 \cdot 0.0022kg}{7.5kg \cdot 1.1m} = 1.43 m/s^2$$

Problem 3 (10 points). A point source emits sound waves with an average power output of **1.3 W**.

a) find the intensity and sound level (in dB) **4.50 m** from the source.
b) find the distance at which the sound level is **95 dB**.

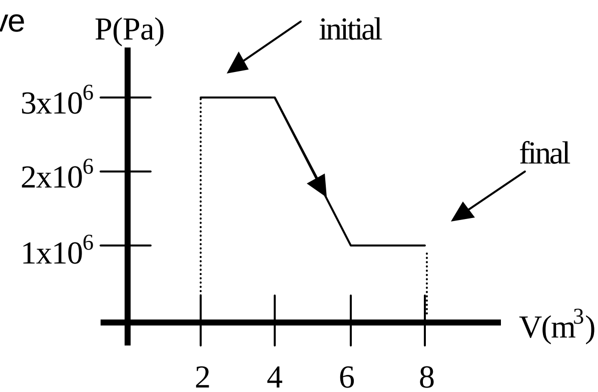
a) the intensity is defined as power/area, and if the source is a point source then the waves are emitted as spherical waves, and have an area given by A=4pr^2 at radius r. The intensity at r=4.5m is then given by $I = \frac{P}{A} = \frac{1.3W}{4p4.5^2} = 0.0051 W/m^2$. The sound level in dB is given by $b = 10 \log I/I_0$ where I_0 is the threshold intensity of 10^{-12} W/m^2, so at 4.5m the sound level is $b = 10 \log 0.0051/10^{-12} = 97.1 dB$.

b) You use the same formula for sound level, but now you have to solve for the intensity where the level is 95dB: $I = I_0 10^{b/10} = 10^{-12}10^{9.5} = 10^{-2.5} W/m^2 = 3.16x10^{-3} W/m^2$ and solving for area using the formula $I = \frac{P}{A}$ gives $A = P/I = 1.3/3.16x10^{-3} = 411.5 = 4pr^2$.
Solving for the radius gives r =5.7m

Problem 4 (15 points). A **1.2 kg** block of copper at **20 °C** is dropped into a large vessel of liquid nitrogen at **71.3K**. How many kilograms of nitrogen boil away by the time the copper reaches **101.0K**? (The specific heat of copper is **0.092 cal/g° C** and the latent heat of vaporization of nitrogen is **48.0 cal/g**.)

The copper's initial temp in Kelvin is 20+273 = 293K. When the copper drops it's temperature to 101.0K, the temperature has decreased by DT = 293-101 = 192K. This temp decrease releases an amount of heat given by Q = mcDT = 1200g·0.092cal/g°C·192 = 21,196.8cal. This heat boils off the nitrogen, given by Q = ML_v so solving for M gives Q/L_v = 21196.8cal/48cal/g = 441.6g = 0.4416kg.

Problem 5 (10 points). In the figure, a fluid expands adiabatically from an initial to a final state. a) Determine the work done by the fluid. b) Calculate the work done on the fluid when it is compressed from final to initial along the same path.



Problem 6

a) work is always area under the P *vs* V curve. This curve has 3 legs: constant volume between 2 and 4, linearly decreasing between 4 and 6, then constant volume between 6 and 8. To find the area under this curve, you can find the area under the 3 legs and add them. For legs 1 and 3 it's easy:
W_1=PDV=3x10^{6}Pa·2m^3=6x10^{6}Joules and
W_3= PDV=1x10^{6}Pa·2m^3=2x10^{6}Joules so they sum to 8x10^{6}Joules. For the 2nd path, the area is given by the sum of a triangle with base 2 and height between 1x10^{6}Pa and 3x10^{6}Pa and the rectangle below the triangle with area 1x10^{6}Pa·2m^3=2x10^{6}Joules.
W_2 = 0.5 base x height (triangle) + 2x10^{6}Joules
= .5·2m^3·(3-1)x10^{6}Joules + 2x10^{6}Joules = 4x10^{6}Joules so the entire work is (6+4+2)x10^6=12x10^{6}Joules.
b) if it goes along the same path backwards, it will take the same amount of energy to compress.

Problem 6 (10 points). A **3.5 liter** vessel contains monatomic gas at **35° C** and **2.50 atm**. Find
a) the total translational kinetic energy of the gas molecules and
b) the average kinetic energy per molecule.

a) you can use the ideal gas law to find the number of particles: $PV = Nk_B T$ and solve for N.

using the formulae for kinetic theory, we have that there is $\frac{1}{2}k_B T$ per degree of freedom of kinetic energy, which means the translational KE per particle is given by

$KE_{trans} = 3\frac{1}{2}k_b T = \frac{3}{2}\frac{PV}{N}$ where the factor of 3 comes from the fact that for monatomic gasses there is KE only in translation, in the 3 dimensions x/y/z. Since we want the total ranslational KE for the entire gas, we multiply by *N* to get

$$E_{trans} = N\frac{3}{2}\frac{PV}{N} = \frac{3PV}{2} = 1.5\cdot 2.5atm\cdot 3.5liter = 13.125liter - atm = 1325.6Joules$$

b) by the same arguments, the average KE per particle is given by

$$KE = \frac{3}{2}k_b T = 1.5\cdot 1.38x10^{-23}J/K\cdot(273+35)K = 6.38x10^{-21}J$$

Problem 7 (10 points). A refrigerator has a coefficient of performance equal to **8.5**. Assuming that the refrigerator absorbs **230J** of energy from a cold reservoir in each cycle, find

a) the work required in each cycle
b) the energy expelled to the hot reservoir.

a) the coefficient of performance is defined as the ratio of the heat taken out of the cold reservoir Q_C per work done W, or $\eta = Q_C/W$. Solving for the work W gives

   $W = Q_C/\eta = 230J/8.5 = 27.1Joules$

b) for a given cycle, the 1st law of thermodynamics says that energy is conserved, which means that the energy into the system ($W+Q_C$) is equal to the energy out of the system Q_H, so we have $Q_H=W+Q_C=230J+27.1J=257.1J$

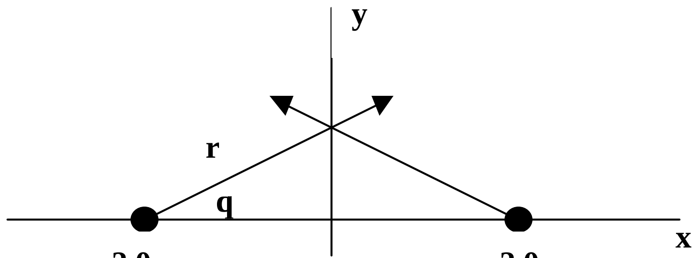
Problem 8 (15 points). Two **5.0mC** point charges are located on the x-axis as in the figure. One is at **x = +2.0 m** and the other is at **x = -2.0m**.

a) determine the electric field on the y-axis at **y = +1.00m** and **x=0**.
b) calculate the electric force on a **–3.00 mC** point charge placed on the y-axis at **y = +1.00m**.

a) since the field due to each charge adds as a vector, we can apply symmetry principles to show that the field at y=+1.0m above the two charges on the y-axis has to point away towards +y, so the contribution from each charge will be given by Esinq where rsin =1m (y-axis location of charge), $r^2 = 2^2 + 1^2 = 5$ E is given by coulomb's law

   $E = \frac{kQ}{r^2} = \frac{9x10^9\cdot 5x10^{-6}}{5} = 9x10^3 N/C$. This gives for the total electric field at the point 1.0m above the x-axis the value $2E\sin\theta = 2E\frac{1}{r} = 2\cdot 9x10^3\cdot\frac{1}{\sqrt{5}} = 8049.8N/C$

b) F=QE gives the force downward (opposite field for a negative charge of –3.00 C) with magnitude $F=3x10^{-6}C\cdot 8049.8N/C=0.024N$



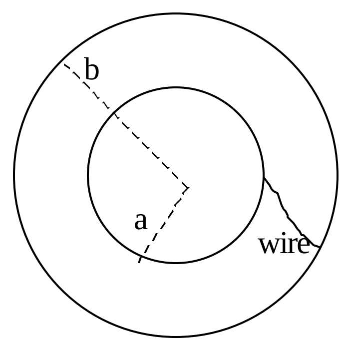
Problem 8

Problem 9 (10 points). Two identical conducting spheres each having a radius of **1.5 cm** are connected by a light **3.0m** long conducting wire. Determine the tension in the wire if **90.0 mC** is placed on one of the conductors. Assume that the surface distribution of charge on each sphere is uniform.

If the charges are shared equally and distributed uniformly, then each sphere will have Q/2 charge, and the force between the conducting spheres will be repulsive and the string will have a tension equal to that force. The distance from the centers will be equal to the sum of the two radii plus the length of the string, or r = 2R+L=3.03m. The force is then pure coulomb: $F = \frac{kQ^2}{r^2} = \frac{9x10^9 \cdot \left(90x10^{-6}/2\right)^2}{3.03^2} = 1.99N$

Problem 10 (10 points). Two concentric spherical conducting shells of radii **a = 0.5m** and **b = 1.0m** are connected by a thin conducting wire as shown in the figure. If the total charge **Q=45.0 mC** is placed on the system, how much charge settles on each sphere?

All the charge will go to the outer sphere. This is because the wire will allow any charge on the inner sphere to run to the outer sphere in order to get as far away from any other charge that it can.

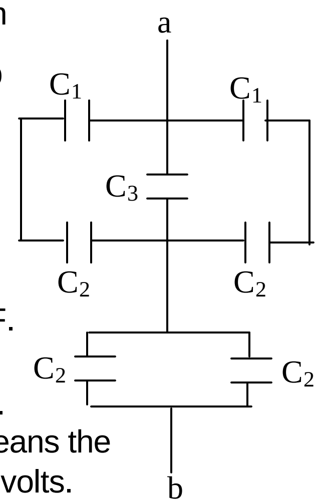


Problem 10

Problem 11 (10 points).

a) Find the equivalent capacitance between points a and b for a group of capacitors connected as shown in the figure if **C_1=2 mF**, **C_2=4 m**F, and **C_3=5 mF**.

b) if the potential difference between points a and b is 1**6V**, what charge is stored on C_3 ?



Problem 11

a) cap's C_1 and C_2 on each side of the upper "rectangle" are in series, with an equivalent capacitance of $\frac{1}{C'_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ so C_{eq}'=4/3 μF. This cap is in parallel with a C_{eq}' on the other side of the "rectangle" and with C_3, with a total equivalent capacitance of C_{eq}''=2·4/3 + 5 = 23/3μF. The bottom 2 C_2 caps are in parallel with each other, forming an equivalent capcitance of 8 μF, and this cap is in series with C_{eq}''=23/3 μF, forming a final equivalent capacitance of $\frac{1}{C_{eq}} = \frac{1}{C_{eq}''} + \frac{1}{8} = \frac{3}{23} + \frac{1}{8} = \frac{47}{184}$ which gives C_{eq}=184/43=4.27 μF.

b) if V=16volts, then the charge on C_{eq} is given by Q=C_{eq}V=4.27x16=68.5 μC. This is the same charge on the equivalent capacitance C_{eq}''=23/3 μF which means the voltage drop across the upper "rectangle" is given by V=Q/C=68.5/(23/3)=8.93volts. Since C_3 is in parallel with the 2 caps labeled C_{eq}' then the voltage across C_3 is 8.93volts, and so the charge on C_3 is given by $Q_3 = C_3V$ = 5x8.93μC = 44.65μC.

Problem 12 (15 points). The heating element of a coffee maker operates at **120V** and carries a current of **3.50A**. Assuming that the water absorbs all of the energy transferred from the heating element, calculate how long it takes to heat **1.5 kg** of water from room temperature **23°C** to the boiling point. Useful units of conversion are 1cal = 4.186Joules.

The power delivered to the heating element will be given by the product P=IV=120·3.5=420Watts. The energy needed to heat water from 23 to 100°C will be given by Q=mcΔT=1500g·1cal/g°C·77°C=115kcal = 483.5kJ. If this energy is delivered over a time Δt, then the power needed will be P=Q/Δt, and solving for Δt=Q/P=483.5kJ/420J/s=1.151ksec = 1151sec = 19.2minutes.

Problem 13 (15 points). One light bulb is marked **'25W 120 V'** (dim) and the other is marked **'100W 120V'** (bright). This means that each bulb converts its respective power to heat and light when plugged into a constant **120V** potential difference. Find

a) the resistance of each bulb

b) How long does it take for **2.20 Coulomb** to pass through the dim bulb?

c) How long does it take for **1.80 Joule** to pass thru the dim bulb?

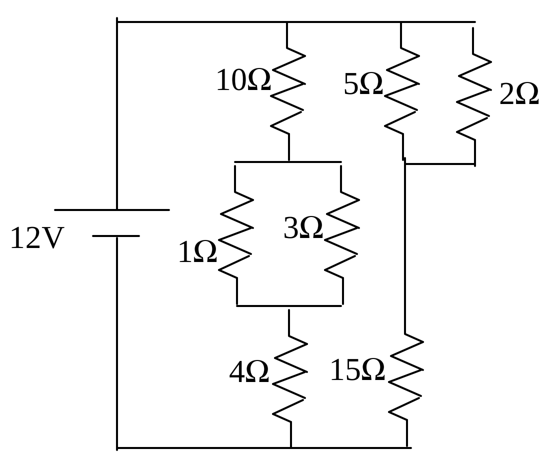
d) Find the cost of running the dim bulb continuously for **30 days** if the electric company sells electricity at **$0.07/kWhr** (kilowatt hour)?

a) each bulb will dissipate power by the rate $P=IV=V^2/R$. solving for $R=V^2/P$. the 25W bulb has a resistance of $R_{25}=120^2/25=576\Omega$ and the 100W bulb has a resistance of $R_{100}=120^2/100=144\Omega$
b) the current in the 25W bulb (the "dim" one) is given by I=V/R=120/576=0.21Amps. current is charge per second, so we want 0.21A=2.2Coulombs/Δt. solving for Δt gives 2.2/0.21=10.56sec.
c) power = energy/time, or P=E/Δt. solving for Δt gives 1.8Joules/25W=0.72sec
d) in 30 days the bulb will use an amount of energy given by $\Delta E=P\cdot\Delta t=25W\cdot 30days\cdot 24h/day=18{,}000Whr$ or 18kWhr. The total cost will then be given by 18kWhr·\$0.07/kWhr=\$1.26

Problem 14 (10 points). In the figure below, the battery voltage is 12 Volts. Calculate
a) the total equivalent resistance in the circuit,
b) the total power dissipated by the entire circuit

a) the 5 and 2Ω resistors are in parallel forming an equivalent resistance of $\frac{1}{R_1}=\frac{1}{5}+\frac{1}{2}=\frac{7}{10}$ or $R_1=10/7=1.43\Omega$. This resistor is in series with the 15Ω resistor, forming an equivalent resistance of $R_{1,eq}$=16.4W. The 1 and 3Ω resistors are also in parallel forming an equivalent resistance of $\frac{1}{R_2}=\frac{1}{1}+\frac{1}{3}=\frac{4}{3}$ or $R_2=3/4=.75\Omega$. This resistor is in series with the 10Ω and 4Ω resistor forming an equivalent resistance of $R_{2,eq}$=10+4+0.75=14.75W. This leaves $R_{1,eq}$ and $R_{2,eq}$ in parallel forming a final equivalent resistance of $\frac{1}{R_{eq}}=\frac{1}{16.3}+\frac{1}{14.75}$ which gives

$R_{eq}=7.77\Omega$.
b) solving for the total current thru the circuit we have $V_B=I_{tot}R_{eq}$ or I_{tot}=12V/7.77W=1.54A. This means that the battery is delivering power $P=I_{tot}V$=1.54A·12V=18.5W. by energy conservation this is the total power dissipated by all the resistors.



Problem 14