HW# 1 -Phys859-Fall 2002 | Prof. Melanie Becker
Due Wednesday, September 25, 2002, by 1.30pm | Room 4125, (301)405-1774
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Problem 1.

A relativistic point particle of mass m moving in Minkowski space has an action S_1 that is proportional to the length of the world-line parametrized by $x^\mu(\tau)$:

$$S_1 = -m \int d\tau \sqrt{|\dot{x}^\mu \dot{x}_\mu|}.$$

The square root makes this action very hard to work with. This problem can be solved by introducing an auxiliary field $e(\tau)$.

a) By deriving the Euler Lagrange equation for $e(\tau)$ show that the action

$$S_2 = -\frac{1}{2} \int d\tau \left(\frac{\dot{x}^2}{e} + em^2 \right),$$

is classically equivalent to S_1 [*5 Points*].

b) Show that the action S_1 is invariant under reparametrizations of the world line by substituting $\tau' = f(\tau)$. Then, show the reparametrization invariance of the action S_2 by checking that it is invariant under the tranformations

$$\delta x = \xi \dot{x}$$
$$\delta e = \frac{d(\xi e)}{d\tau}.$$

[*5 Points*].

c) Obtain the equation of motion for x and e for the action S_2. Use the reparametrization invariance to set $e = 1$ after differentiating S_2 with respect x and e. What is the interpretation of the resulting equations? [*5 Points*].

Problem 2.

Show that the p-brane action $\int d\mu_{p+1}$ described in class is invariant under diffeomorphisms [*5 Points*].

Problem 3.

Show that if a "cosmological constant" term is added to the string world-sheet action it leads to inconsistent classical equations of motion [*10 Points*].

Problem 4.

a) Find the mode expansion for an open string with Dirichlet boundary conditions $X^\mu(0,\tau) = X_0^\mu$ and $X^\mu(\pi,\tau) = X_\pi^\mu$ [*5 Points*].

b) Find the mode expansion for an open string with the Dirichlet boundary condition $X^\mu(0,\tau) = X_0^\mu$ and the Neumann boundary condition $X'^\mu(\pi,\tau) = 0$ [*5 Points*].

Problem 5.

Verify the Virasoro algebra for the bosonic string. In particular, show that the central charge c equals the space-time dimension D [*10 Points*].