Why is it so hard to measure the strength of gravity?

Stephen M. Merkowitz

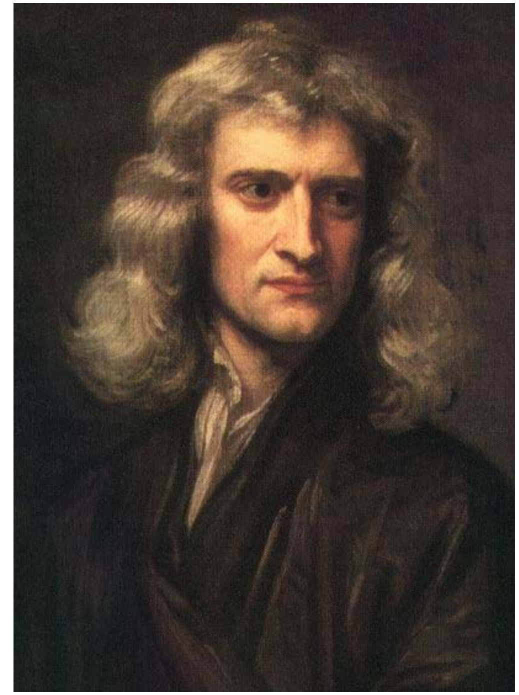
NASA/GSFC

March 8, 2007

Newton's Law of Gravity

"there is a power of gravity pertaining to all bodies, proportional to the several quantities of matter which they obtain...

the force of gravity towards the several particles of any body is inversely as the square of the distances of places from the particles" - Isaac Newton



$$F_G = G\frac{Mm}{r^2}$$

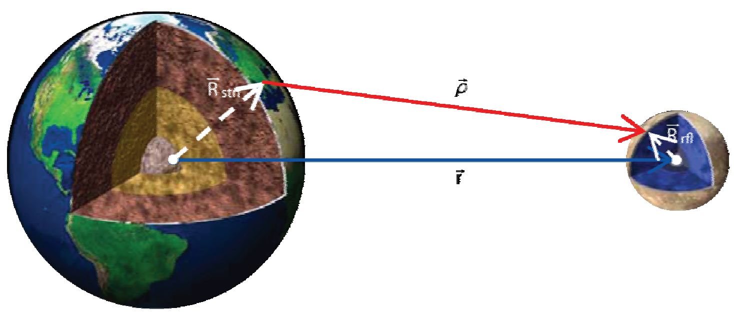
- What is the quantity of matter?
- What is the distance dependence?
- What is the strength of gravity?

What is the quantity of matter?

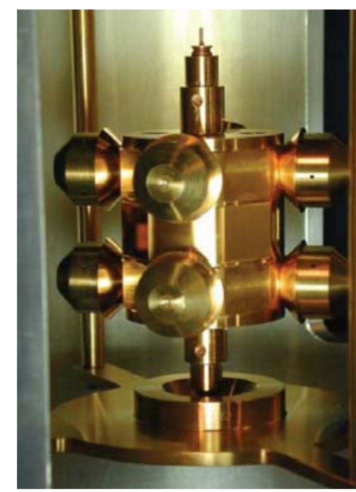
Equivalence Principle Tests

$$\text{Inertial mass} \stackrel{?}{=} \text{Gravitational mass}$$

Lunar Laser Ranging

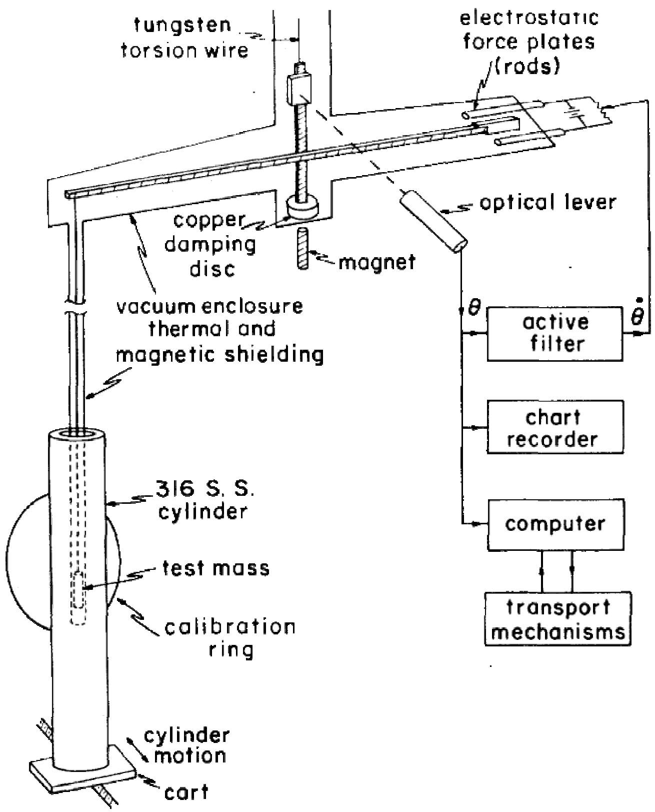


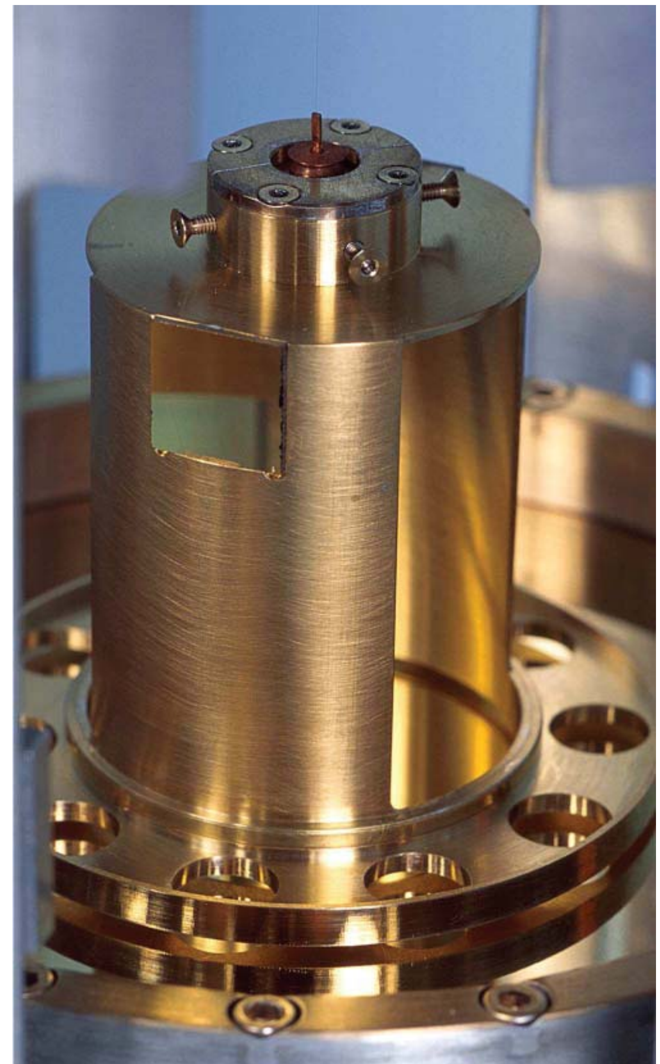
Torsion Pendulum Tests



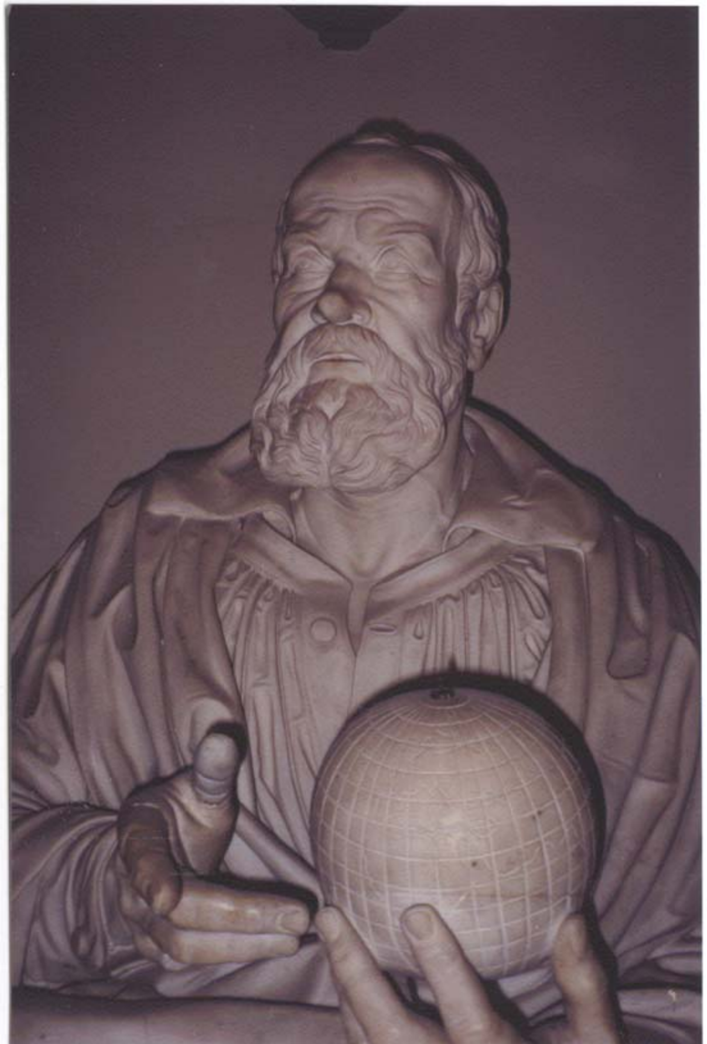
What is the distance dependence?

Inverse Square Law Tests





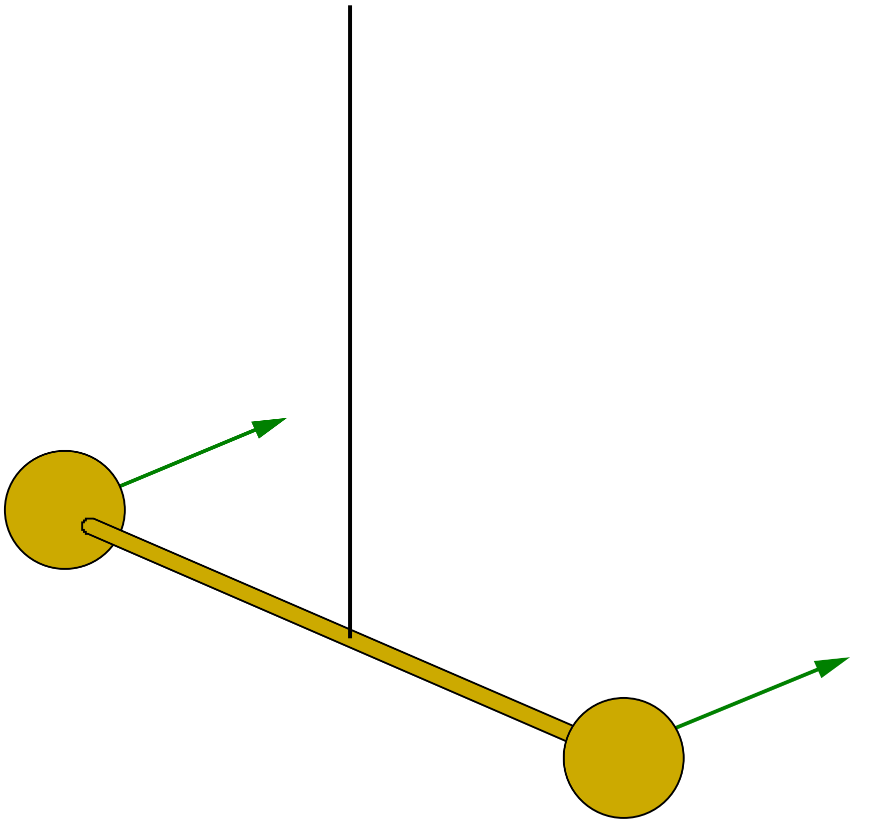
What is the strength of gravity?



- G is a fundamental physical constant
- Planetary motion - used to weight the planets and the sun
- Hydrostatic equilibrium for stars
- Planck scale
- May play a central role in unification at high energies (short distances)
- Fundamental constant in metrology?

Why is it so hard to measure the strength of gravity?

- Absolute measurement dependent on mass, length, & time.
- Gravitational interaction ~1000 times smaller than electromagnetic interaction.
- Gravitational force cannot be shielded.
- G has no known dependence on another fundamental constants.
- Measuring instrument of choice, torsion pendulum, is subject to a variety of parasitic couplings and systematic effects.



Henry Cavendish (1798)

"Experiments to determine the density of the Earth"

Density of Earth = 5.448 x Density of Water

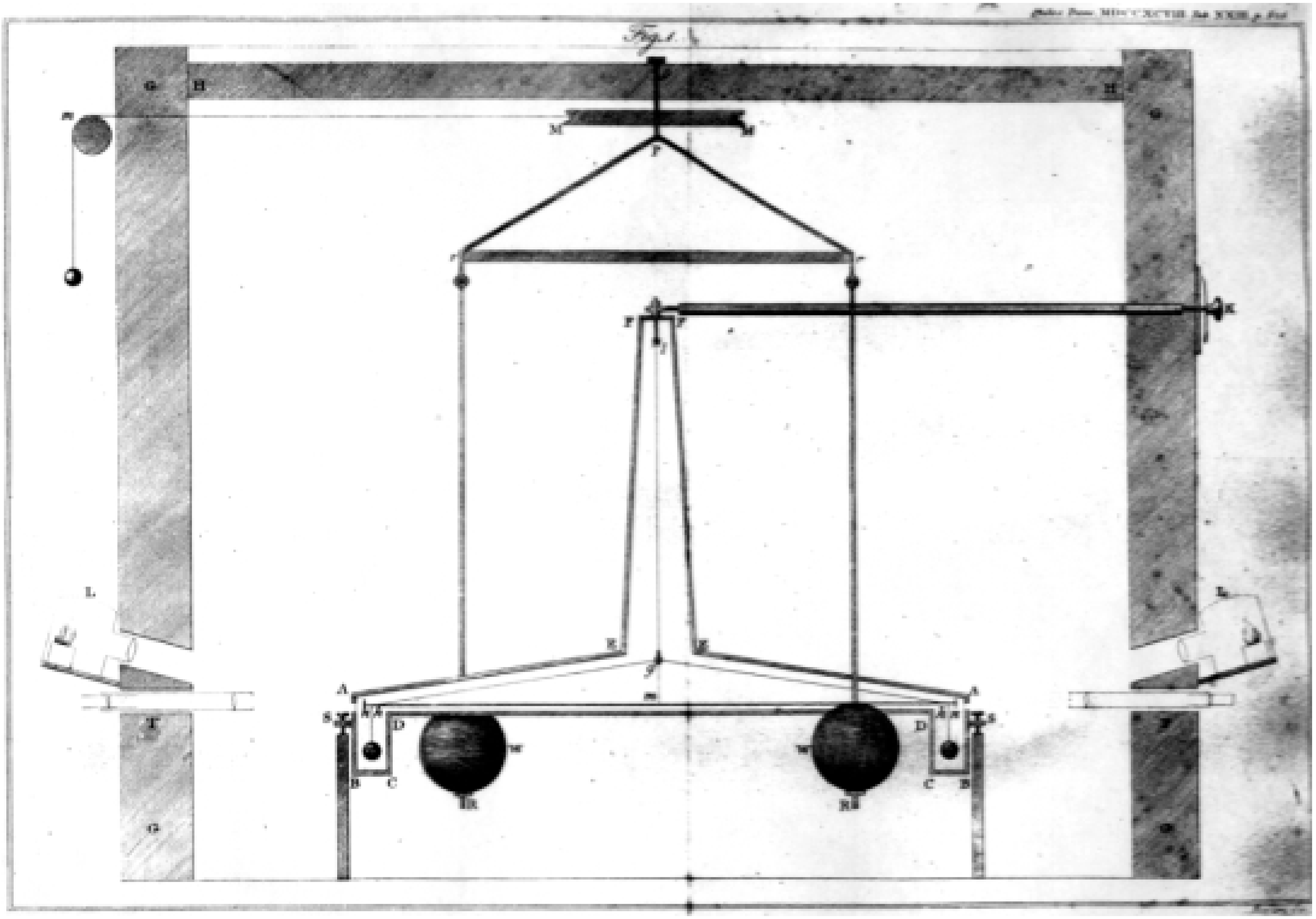
$$G = \frac{3g}{4\pi R_{\oplus}\rho_{\oplus}}$$

Cornu & Baille (1873) used G as scale factor

$$G = (6.754 \pm 0.05) \times 10^{-11} \mathrm{Nm}^2/\mathrm{kg}^2$$

I. Falconer, Measur. Sci. Tech. **10**, 470 (1999).

Cavendish Experiment

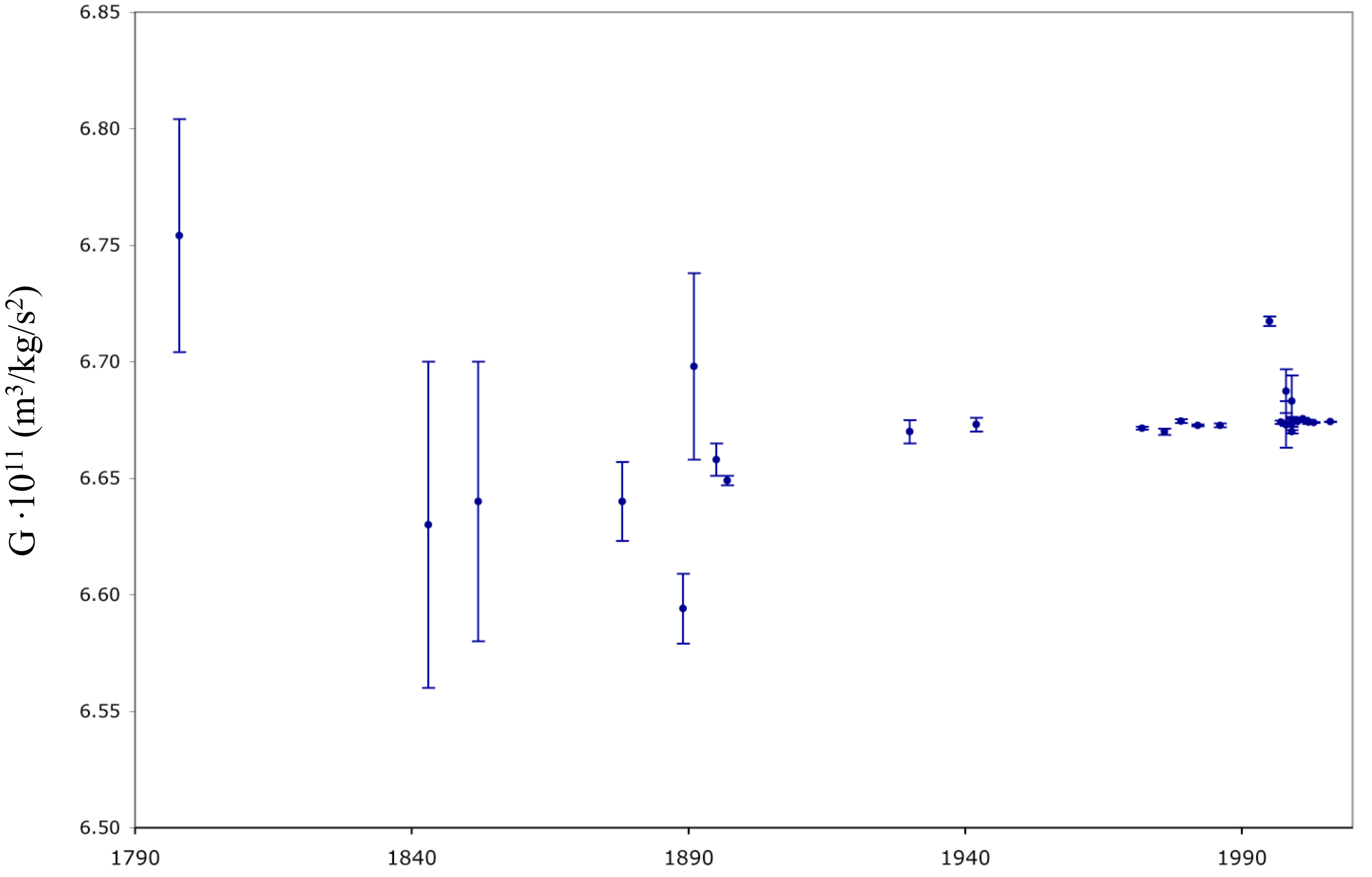


"As the force with which the balls are attracted by these weights is excessively minute, not more than 1/50 000 000 of their weight, it is plain, that **a very minute disturbing force will be sufficient to destroy the success of the experiment**"

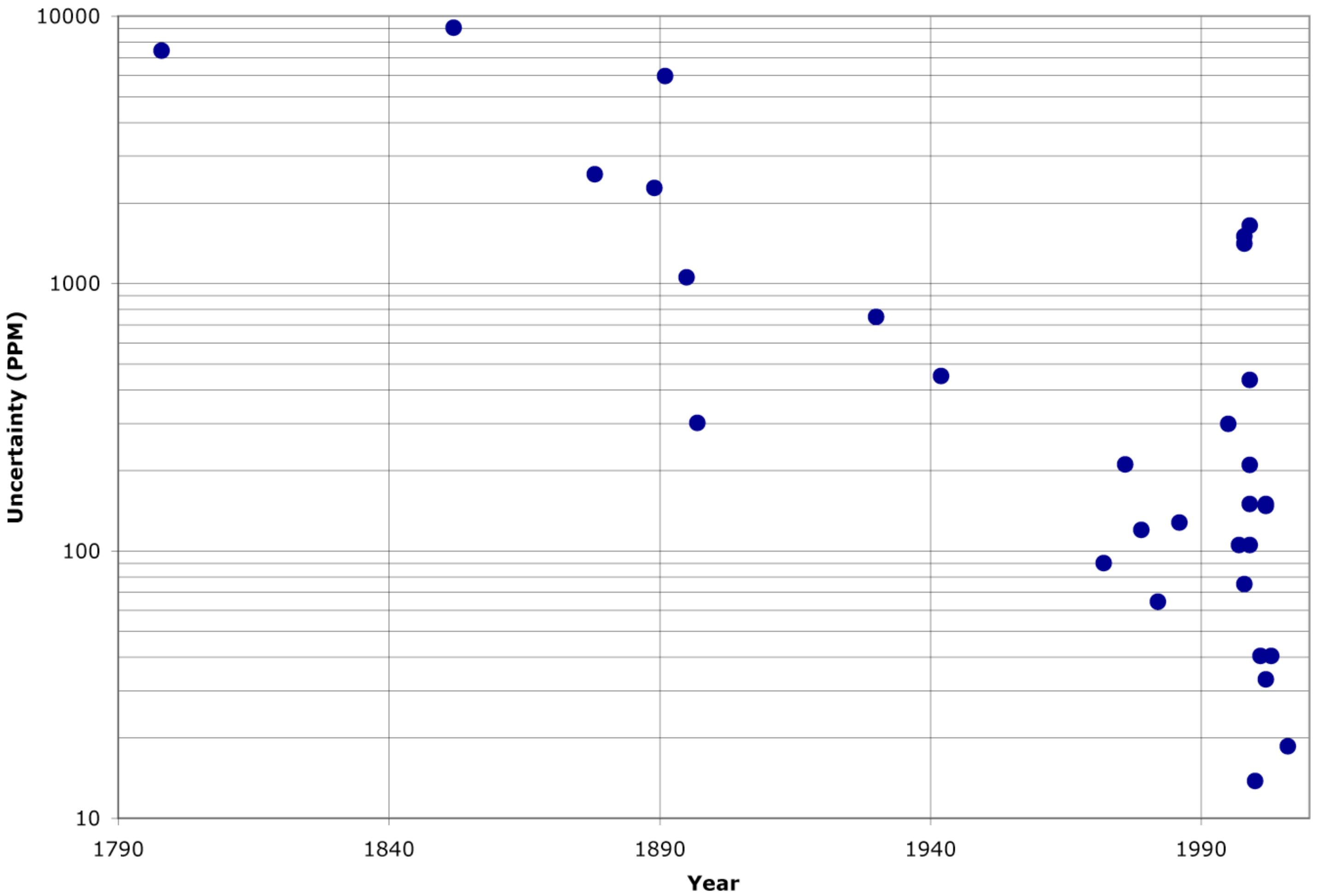
- Henry Cavendish

Apparatus by John Michell

G Measurements

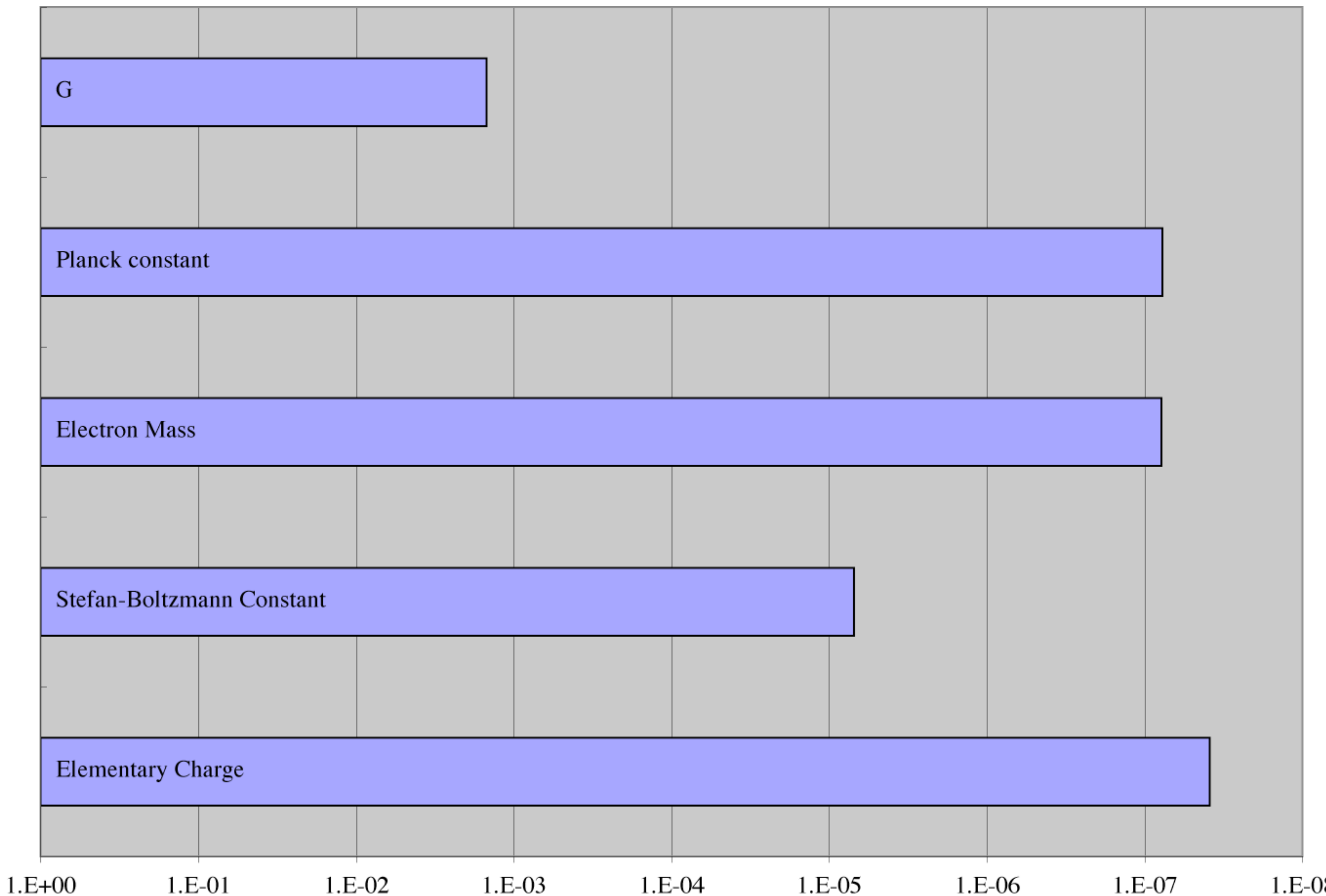


Measurement Error

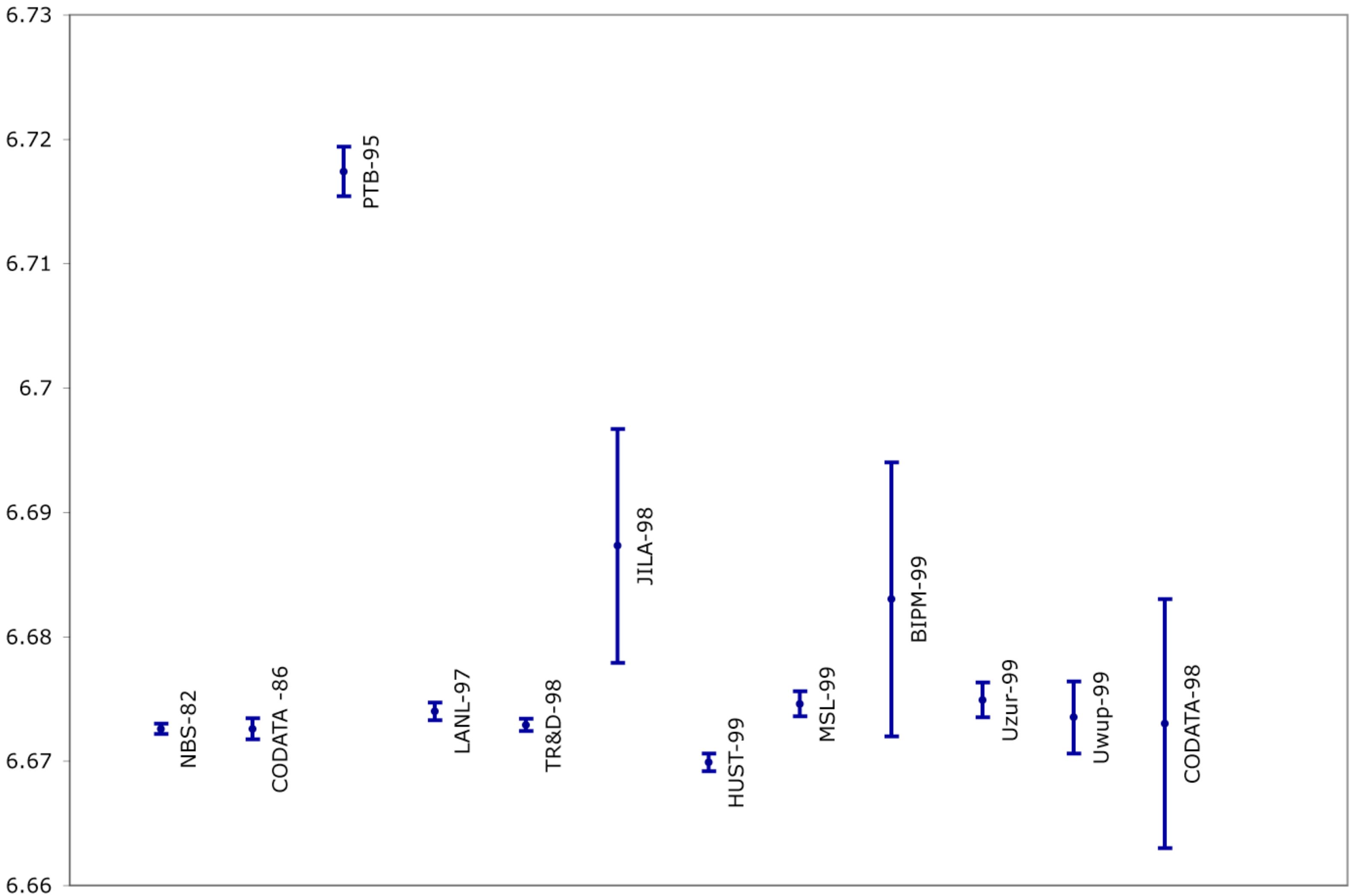


Uncertainty of G has increased only by about an order of magnitude per century!

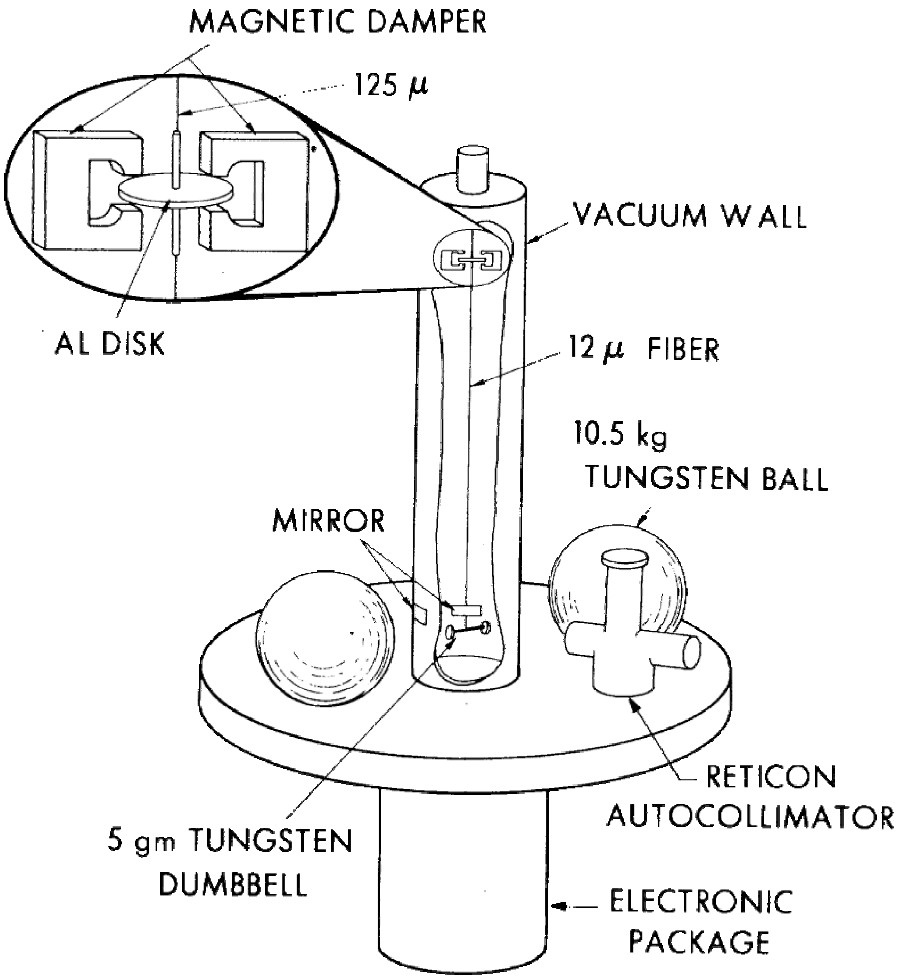
1998 Relative Situation



The Big G Controversy



NBS G Measurement



Time of swing method

$$\omega^2 = \left(K_f + K_g\right)/I$$

$$K_g = \frac{d^2U}{d\varphi^2}$$

$$G = \frac{\Delta\left(\omega^2\right)I}{k_g}, \qquad k_g = K_g/G$$

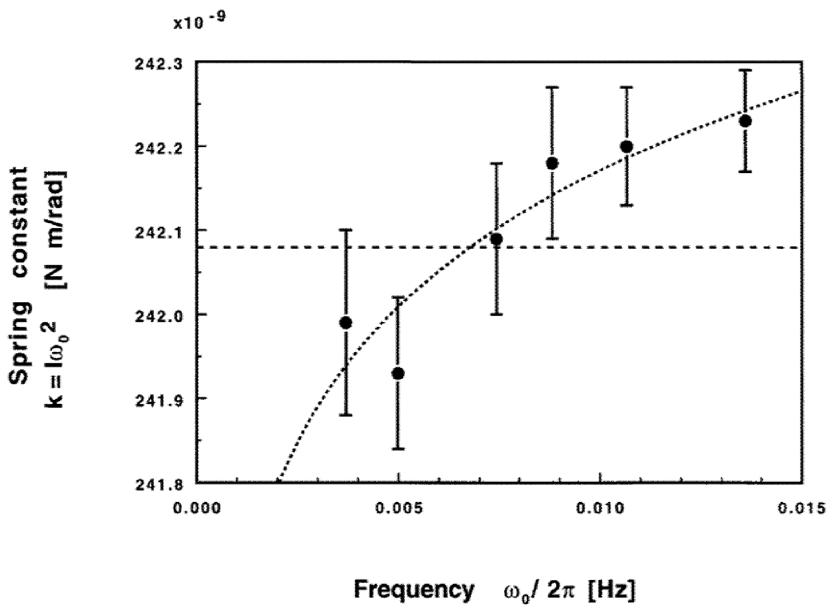
Source	PPM
Position of large masses	10
Mass of large masses	1
Length of small mass	22
Thickness of small mass	36
Density of small mass	6
Moment of inertia of mirror	23
Frequency difference	40
Total	64

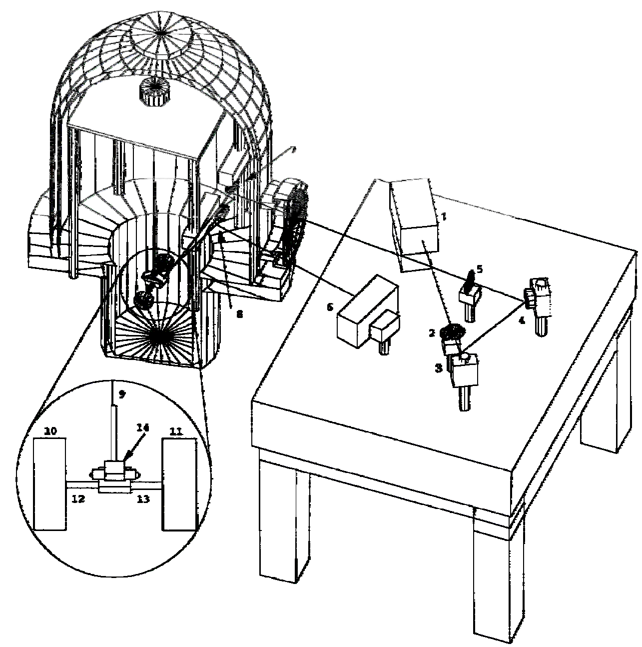
$$G = (6.6726 \pm 0.0005) \times 10^{-11}\,\mathrm{Nm}^2/\mathrm{kg}^2$$

G. G. Luther and W. R. Towler, Phys. Rev. Lett. **48**, 121 (1982).

Limitations of Time of Swing Method

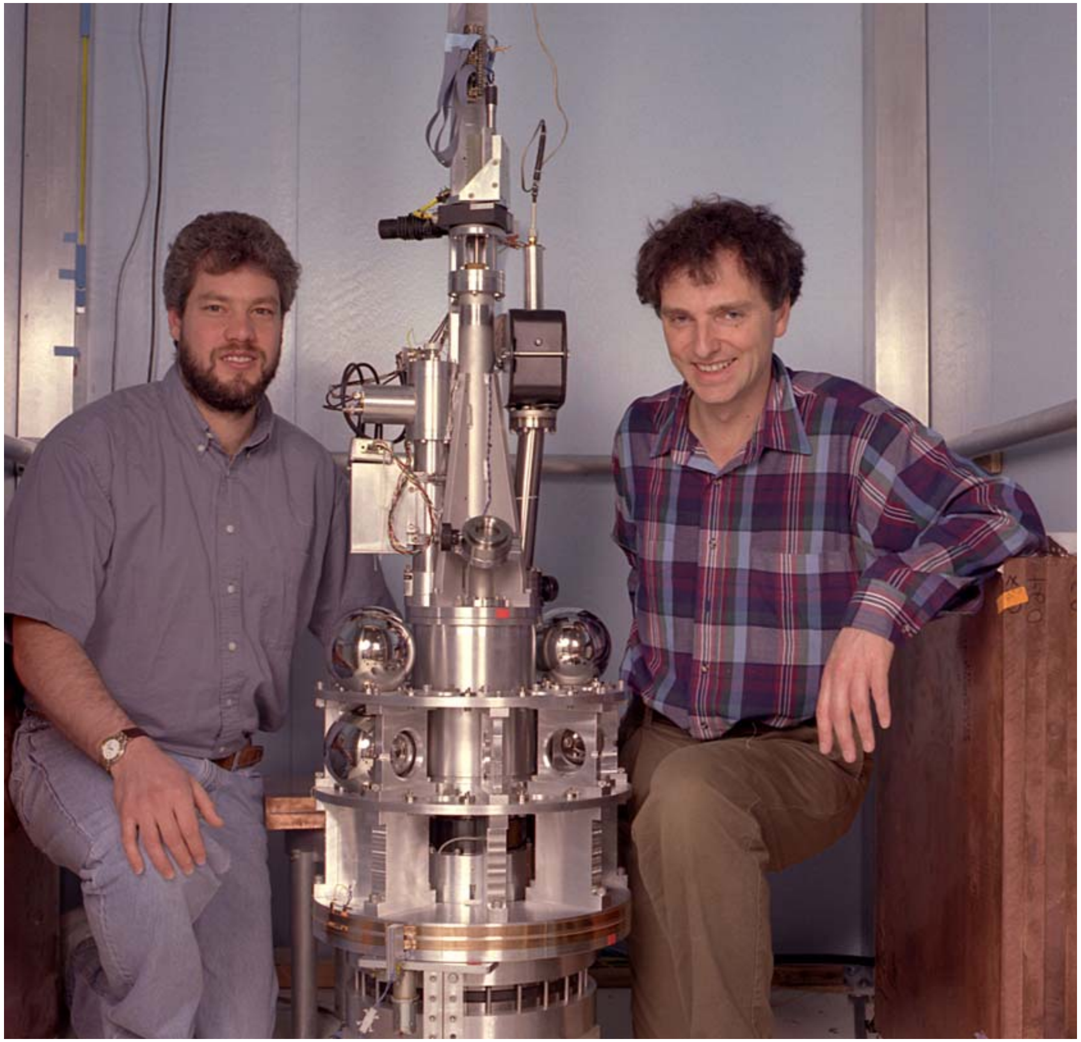
- Changes in environment
- Changing environmental mass distribution
- Uncertainty of pendulum mass distribution
- 1/f noise
- Anelastic fiber properties (Kuroda effect)





K. Kuroda, Phys. Rev. Lett., **75**, 2796 (1995).
S. Matsumura *et al.*, Phys. Lett. A **244**, 4 (1998).

University of Washington G Measurement



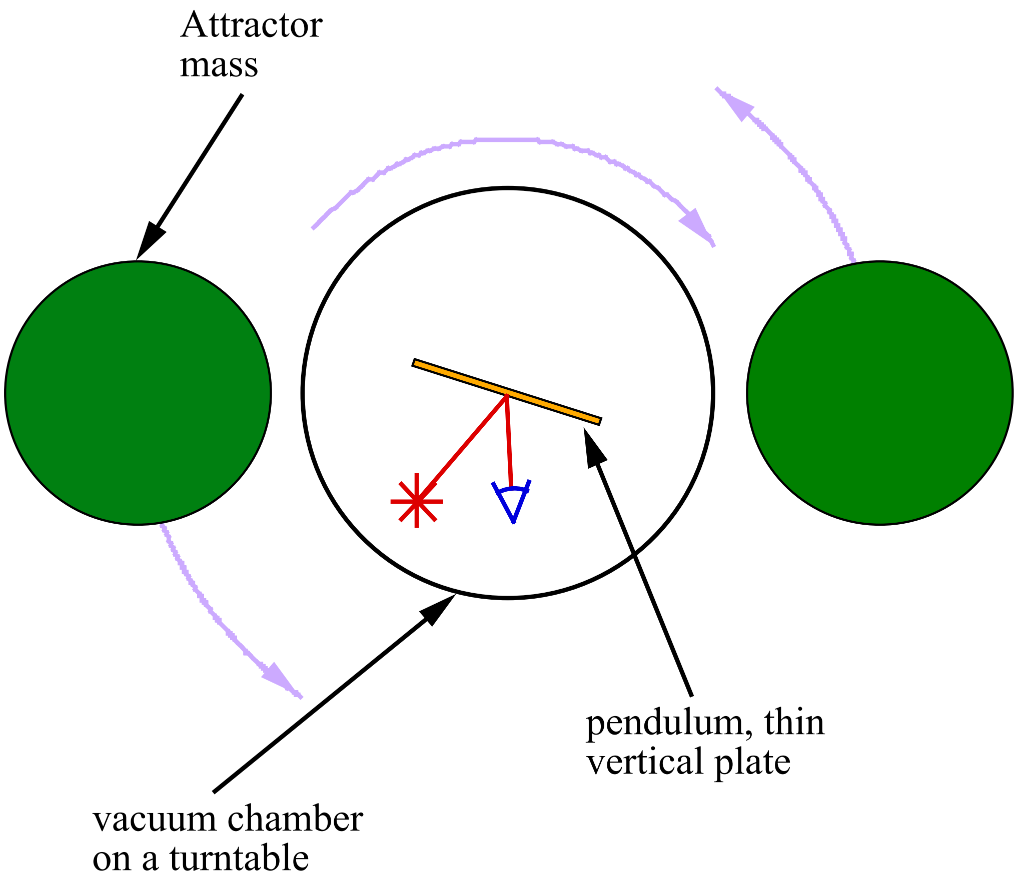
Source	PPM
Pendulum Width	0.4
Pendulum Thickness	4.0
Attractor Diagonal	7.1
Ball-bar Calibration	1.4
Attractor Vertical	5.2
Sphere Diameter	2.6
Attractor Temperature	6.9
Sphere Mass	0.4
Air Humidity	0.5
Remaining Twist	0.3
Magnetic Field	0.6
Rot. Temperature Grad.	0.4
Time Base:	0.1
Data Reduction	2.0
Statistical Error	5.8
Total	**13.7**

$$G = (6.674255 \pm 0.000092) \times 10^{-11} \mathrm{Nm}^2/\mathrm{kg}^2$$

J. H. Gundlach and S. M. Merkowitz, Phys. Rev. Lett. **85**, 2869 (2000)

Method

- Start pendulum turntable at constant rate
- Turn on acceleration feedback so torsion fiber does not twist
  - No Kuroda effect
  - Angular acceleration transferred to turntable
  - Angle encoding absolutely calibrated
- Attractor mass on different turntable rotating at constant difference speed



Gravitational acceleration of a torsion balance

$$\alpha(\varphi) = \sum_{\ell,m} \alpha_{\ell m} = -\frac{4\pi G}{I} \sum_{\ell=1}^{\infty} \frac{1}{2\ell+1} \sum_{m=-\ell}^{+\ell} m \overline{q}_{\ell m} Q_{\ell m} e^{im\varphi}$$

$$\overline{q}_{\ell m} = \int \rho_{\text{pend}}(r) r^{\ell} Y_{\ell m}{}^{*}(\hat{r}) d^3 r$$

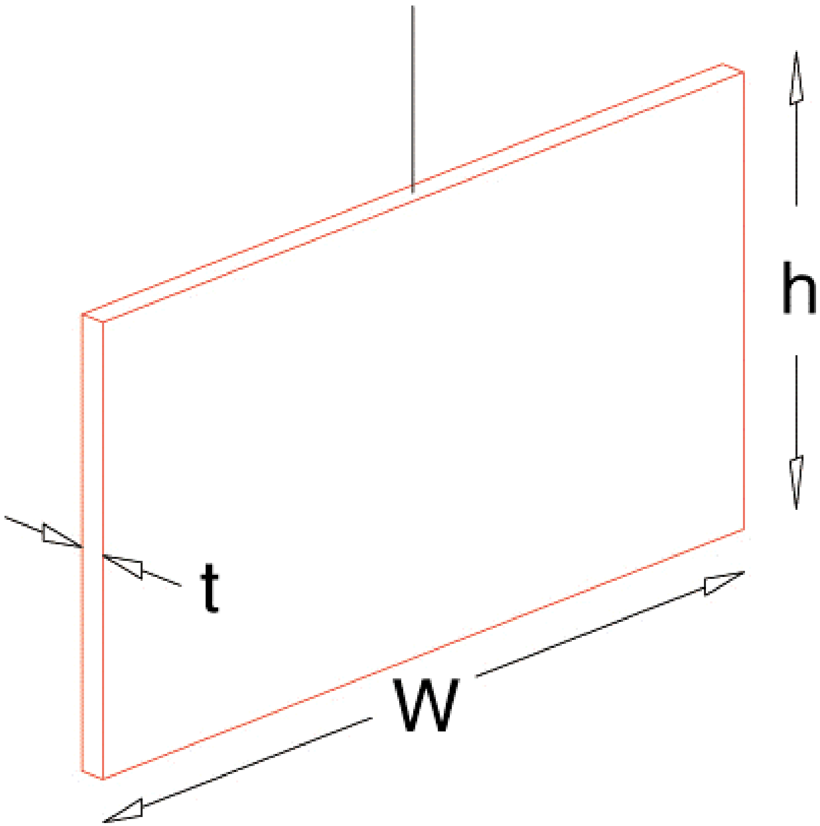
$$Q_{\ell m} = \int \rho_{\text{source}}(r) r^{-(\ell+1)} Y_{\ell m}(\hat{r}) d^3 r$$

Biggest component is α_{22}

$$\alpha_{22} = -\frac{16\pi}{5} G \frac{q_{22}}{I} Q_{22} \sin 2\varphi$$

Signal periodic at twice per revolution

Thin Plate Pendulum



$$\alpha_{22} = -\frac{16\pi}{5} G \frac{q_{22}}{I} Q_{22} \sin 2\varphi$$

$$\frac{q_{22}}{I} = \frac{\int \rho(\vec{r}_p) Y_{22}(\theta_p, \phi_p) r_p^2 d^3 r_p}{\int \rho(\vec{r}_p) \sin^2 \theta_p r_p^2 d^3 r_p} \xrightarrow{\text{2D}} \sqrt{\frac{15}{32\pi}}$$

$$\alpha_{22} = -\sqrt{\frac{24\pi}{5}} G Q_{22} \sin 2\varphi$$

Finite thickness plate: $\frac{q_{22}}{I} = \frac{w^2 - t^2}{w^2 + t^2} \sqrt{\frac{15}{32\pi}}$

Minimize other gravity moments

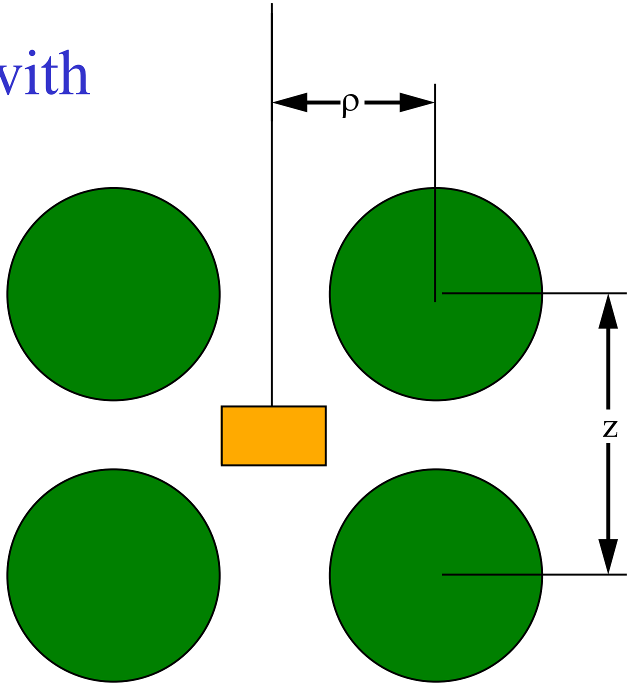
$\alpha_{\ell 2}$ ℓ=odd vanish by symmetrical design of pendulum & source

α_{42} q_{42} vanishes for rectangular plate with

$$h^2 = \frac{3}{10}\left(w^2 + t^2\right)$$

Q_{42} vanishes for vertical sphere separation of

$$z = \sqrt{\frac{2}{3}}\rho$$

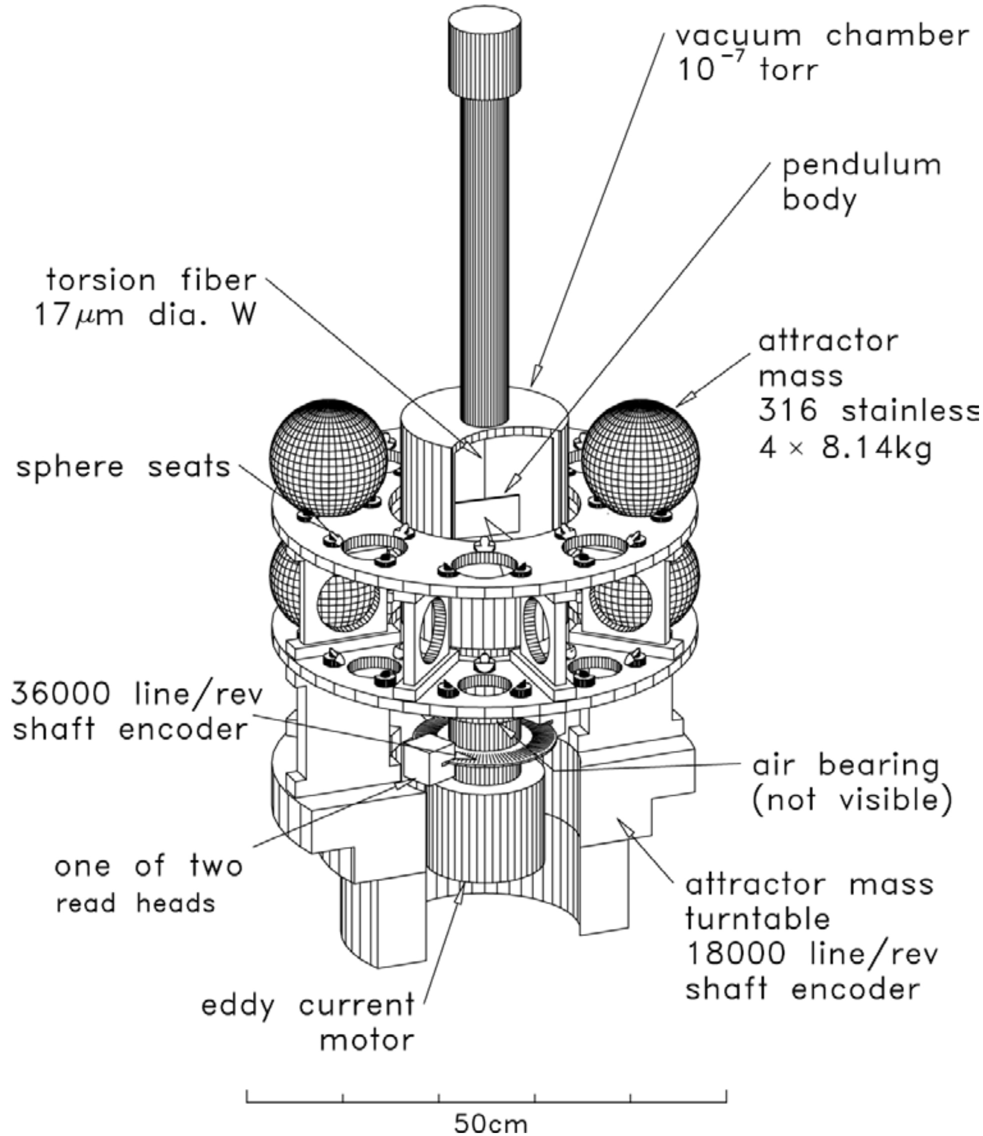


α_{62}, α_{82} Small and calculable

Higher terms small, fall of as $\left(\frac{r}{R}\right)^{\ell-2}$

r=pendulum radius
R=attractor mass radius

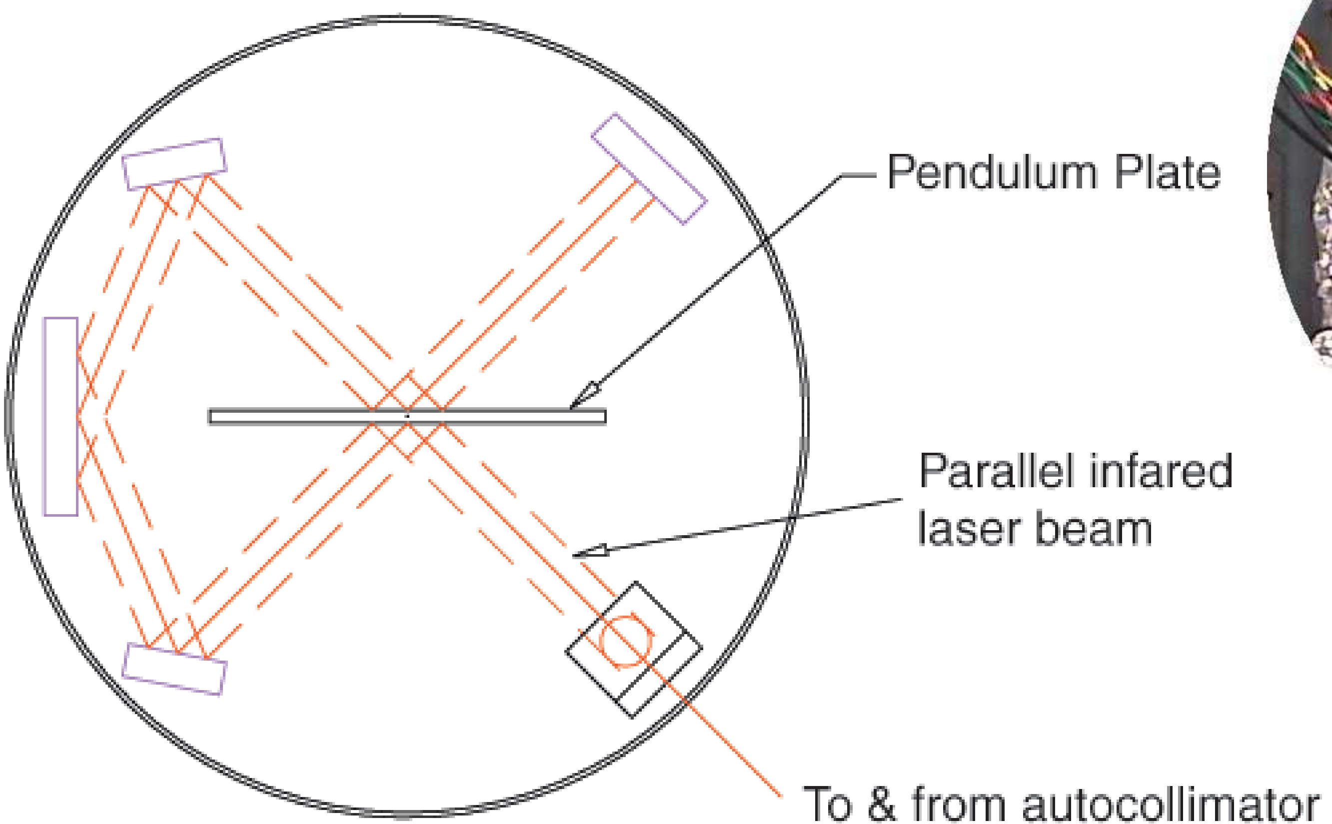
Apparatus

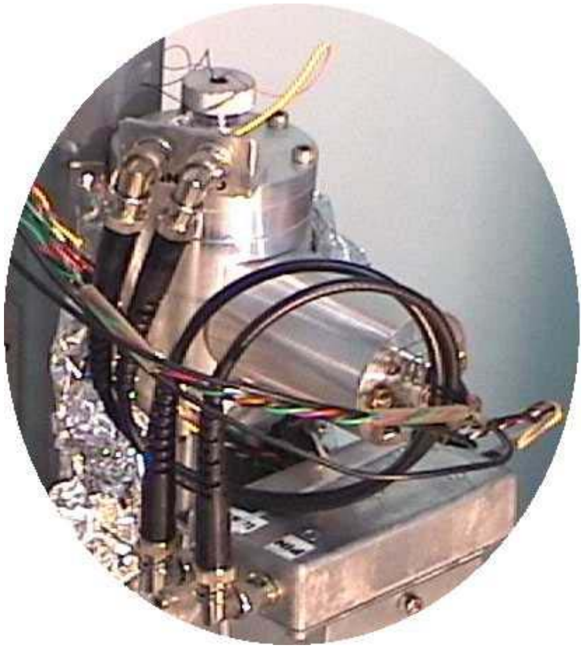

vacuum chamber
10^{-7} torr
pendulum body
torsion fiber
17μm dia. W
attractor mass
316 stainless
4 × 8.14kg
sphere seats
36000 line/rev
shaft encoder
air bearing
(not visible)
one of two
read heads
attractor mass
turntable
18000 line/rev
shaft encoder
eddy current
motor
50cm

Big G Apparatus

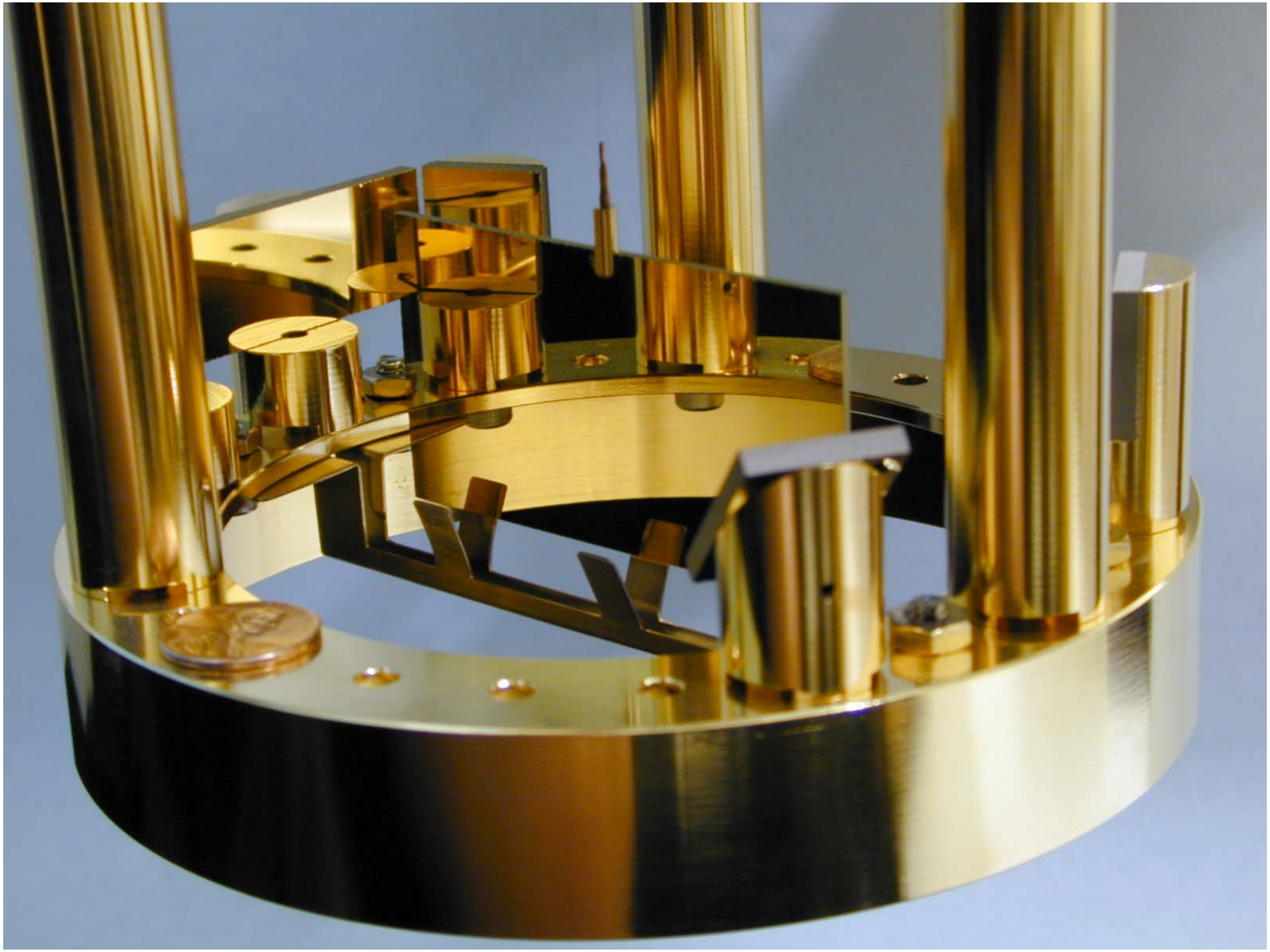
Pendulum position

- 4 bounces off pendulum plate gives angle amplification of 8
- Insensitive to other rotations

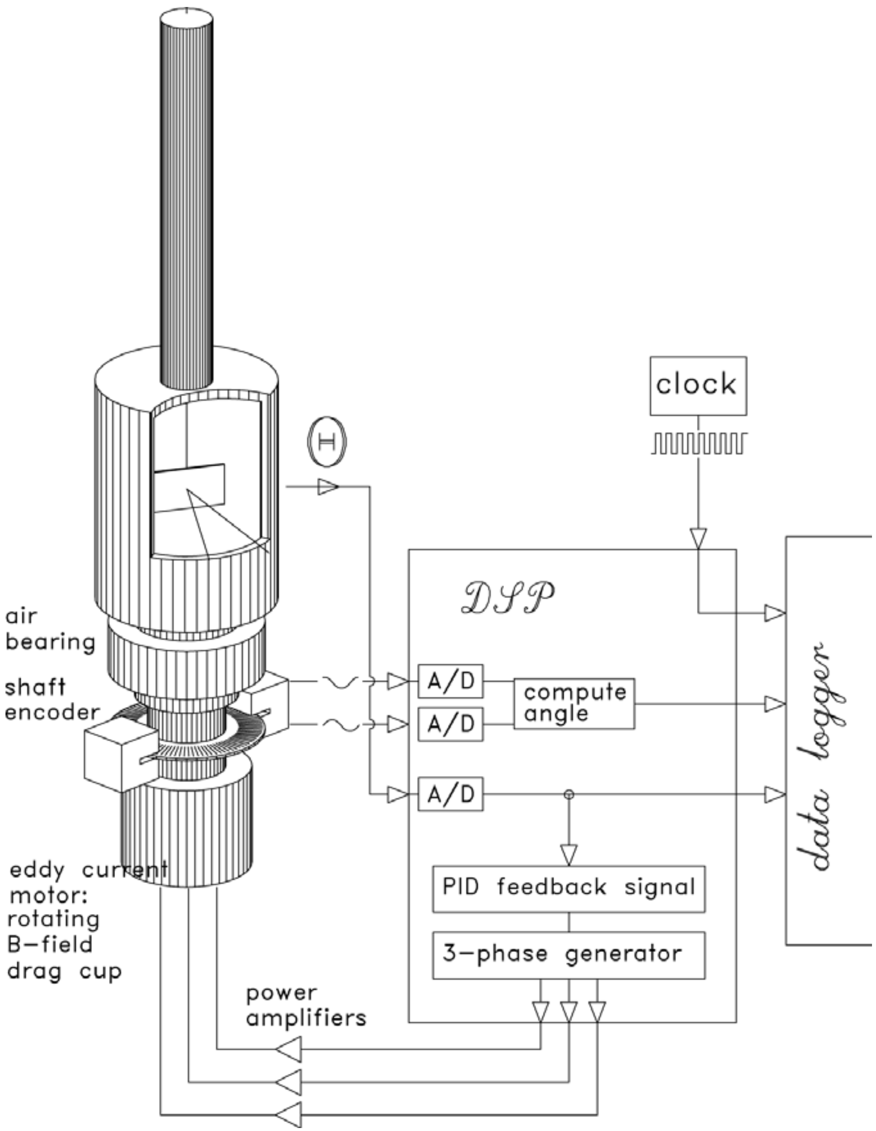


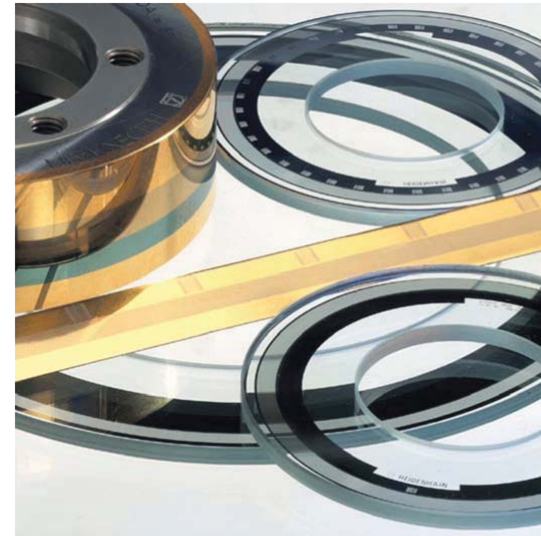


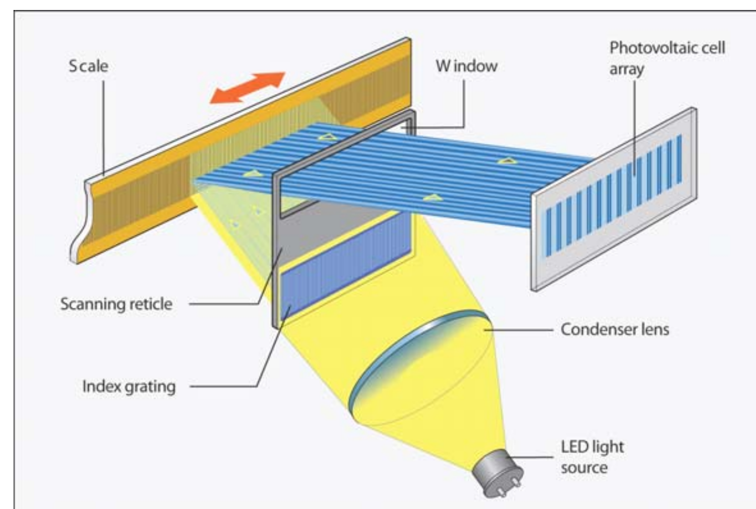
Pendulum



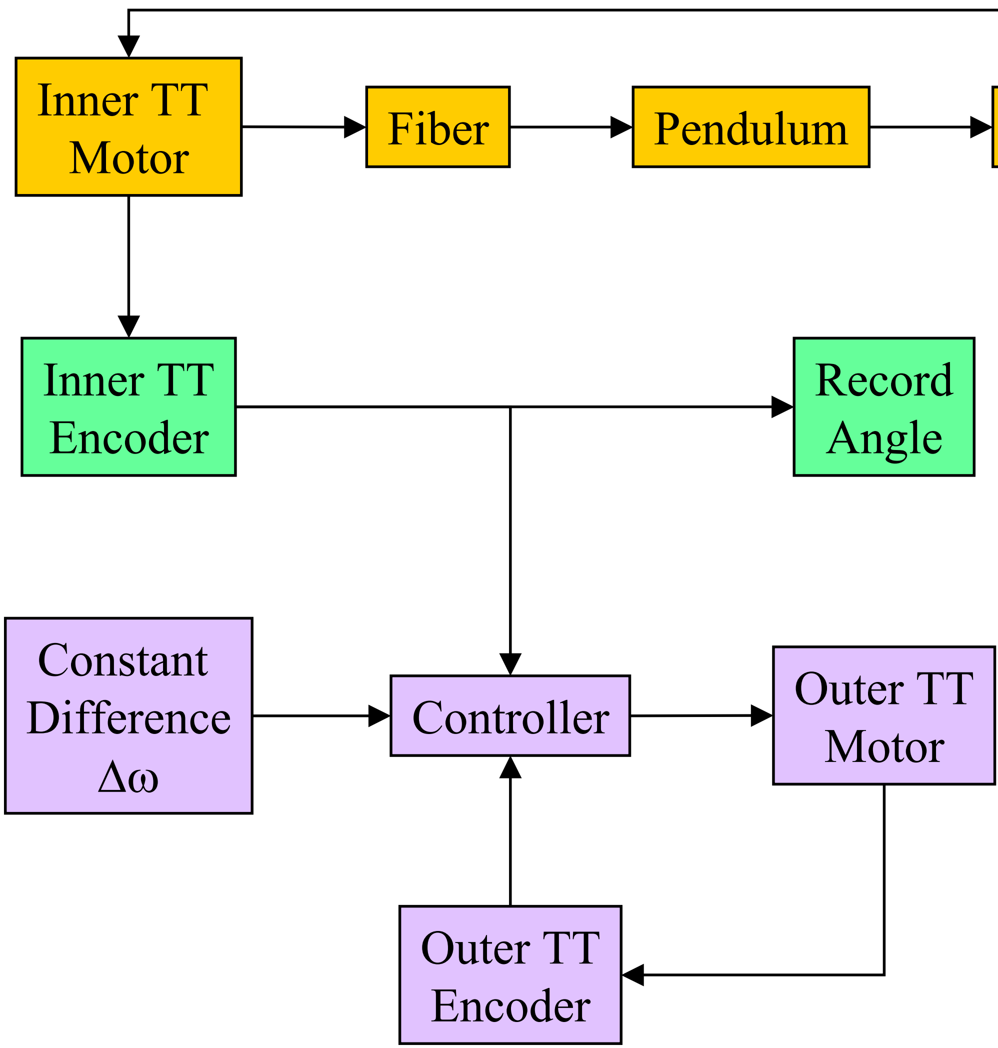
Turntable Readout

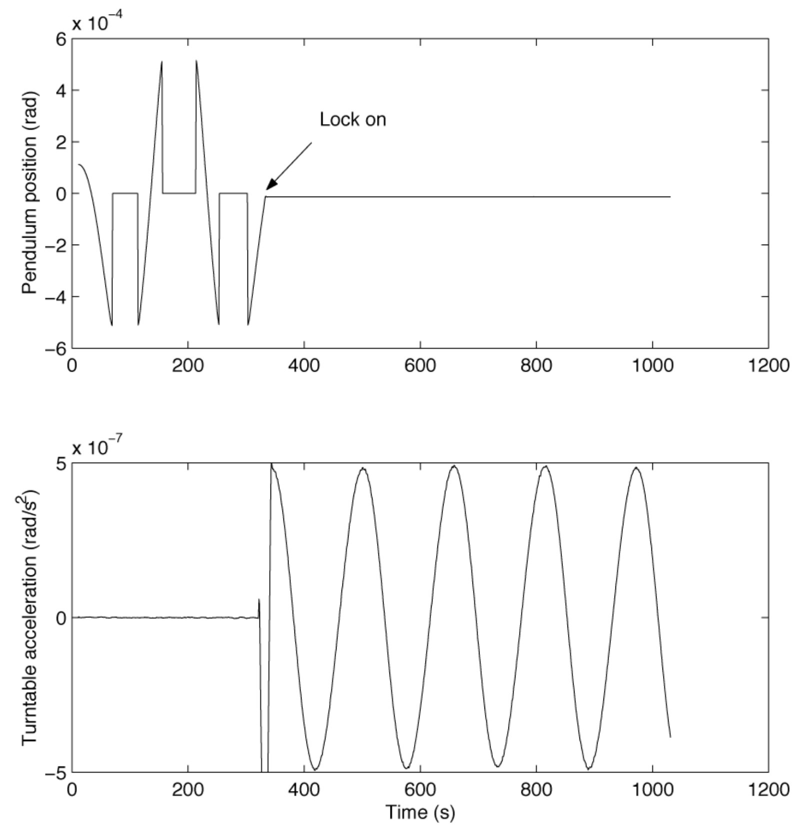




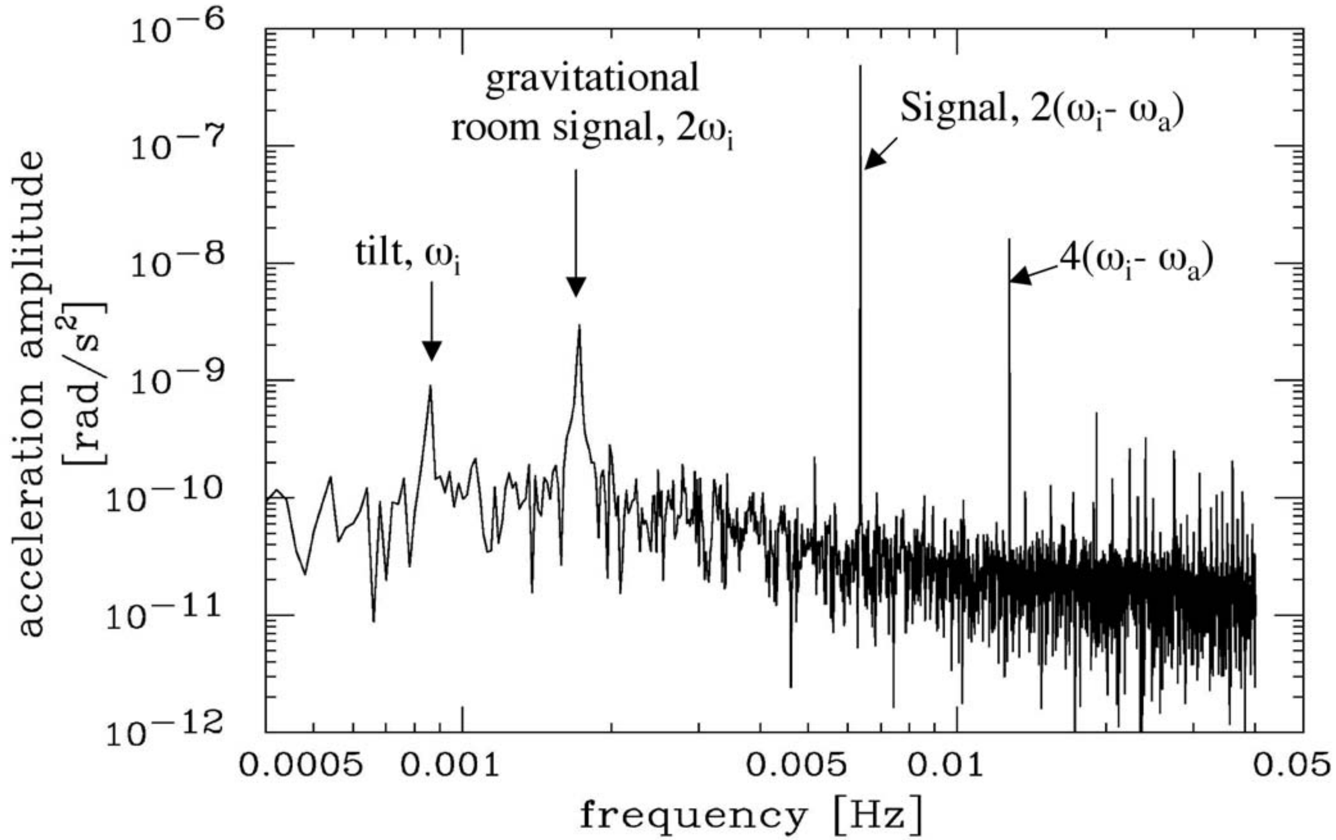


Pendulum-Turntable Feedback

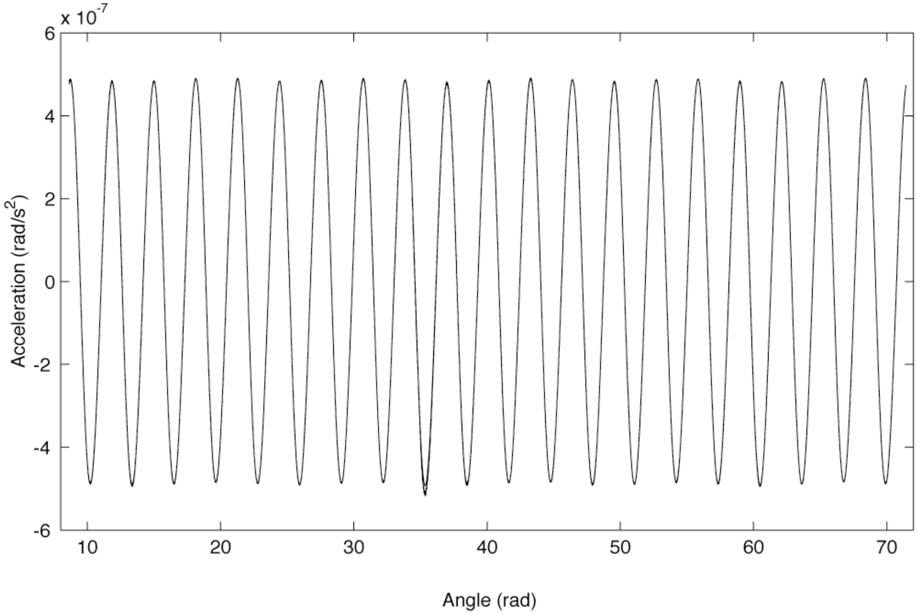


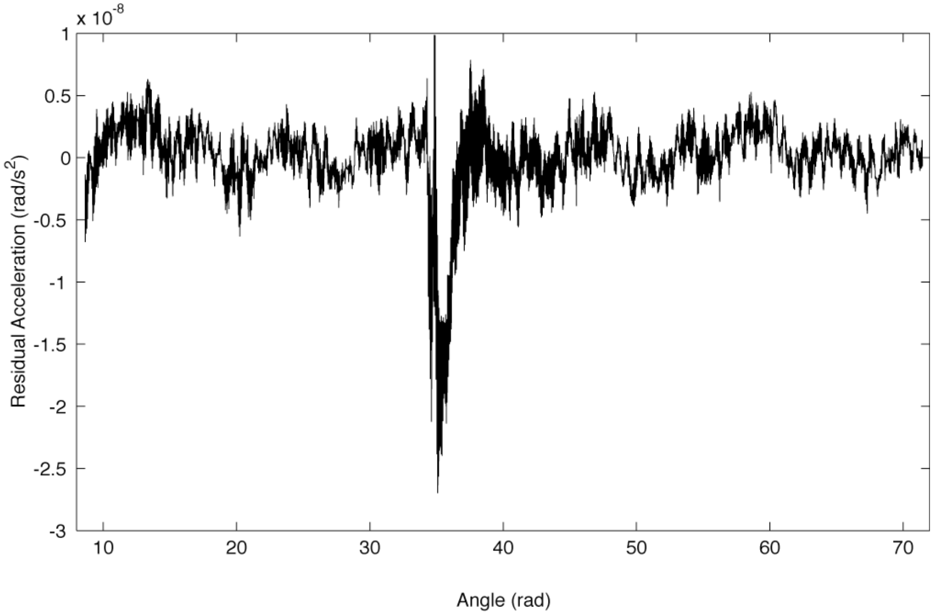


Signal Spectrum



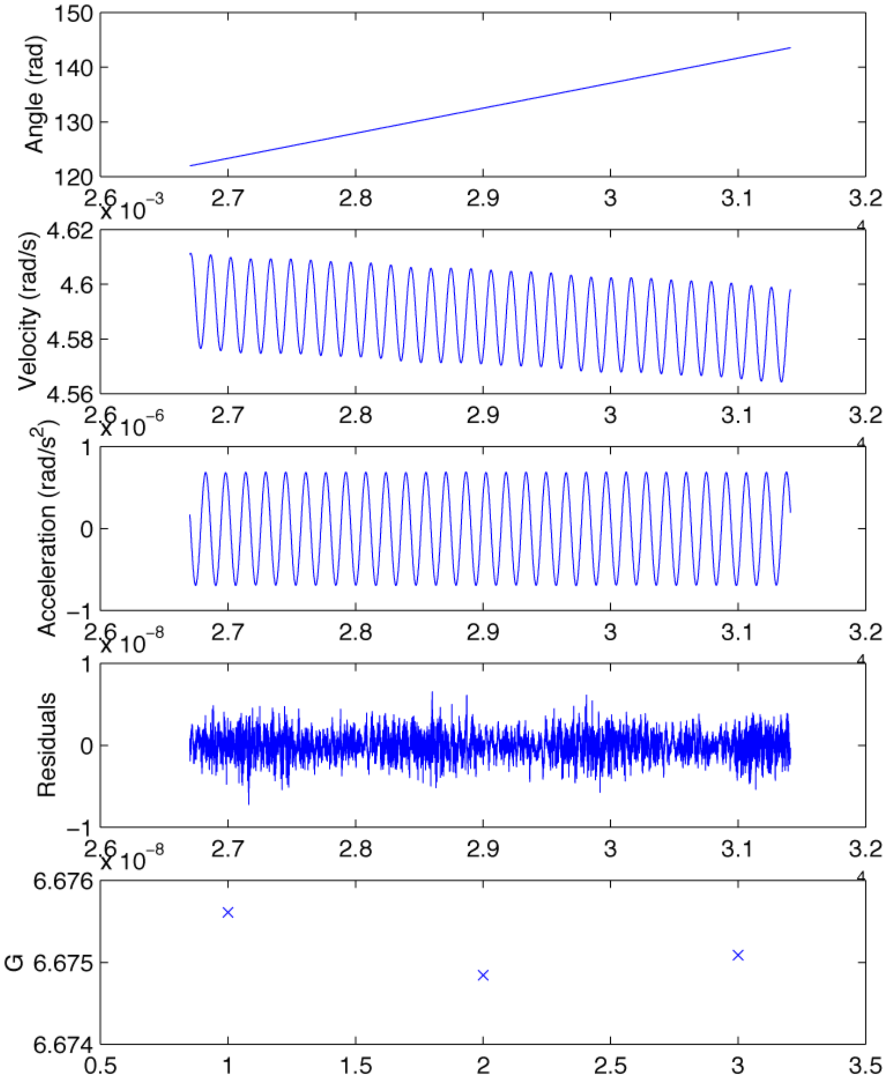
Bad Data





Implosion of Kingdome 4 miles away

Fitting for G

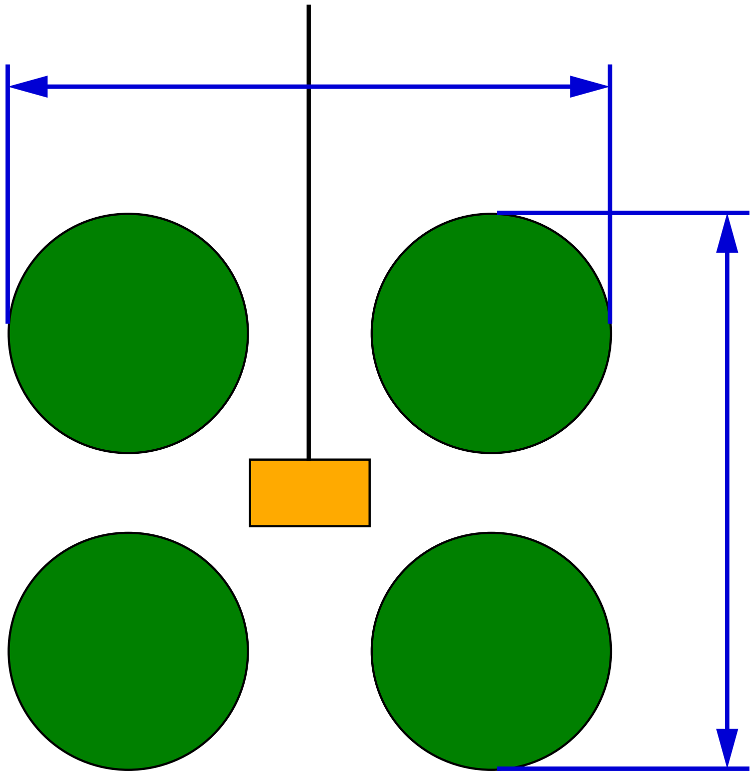


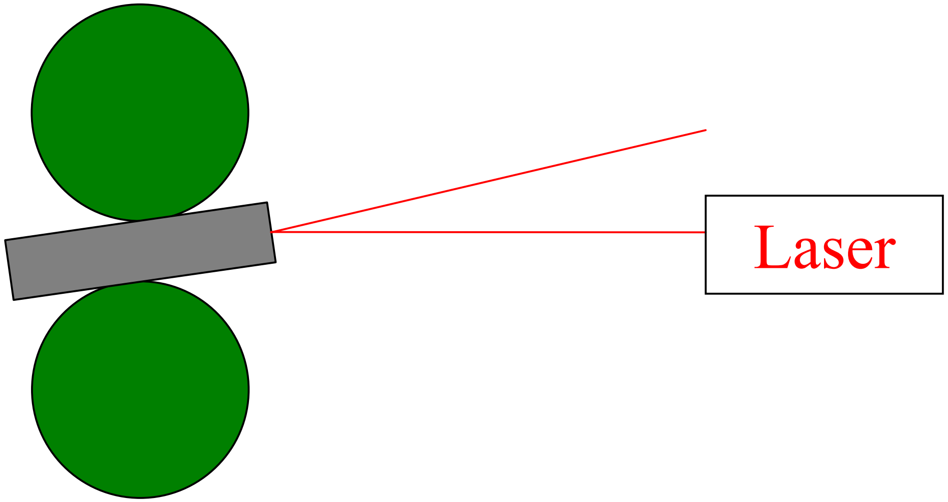
$$\alpha_{22}(\phi) = -\sqrt{\frac{24\pi}{5}}\, G Q_{22} \sin 2\phi$$

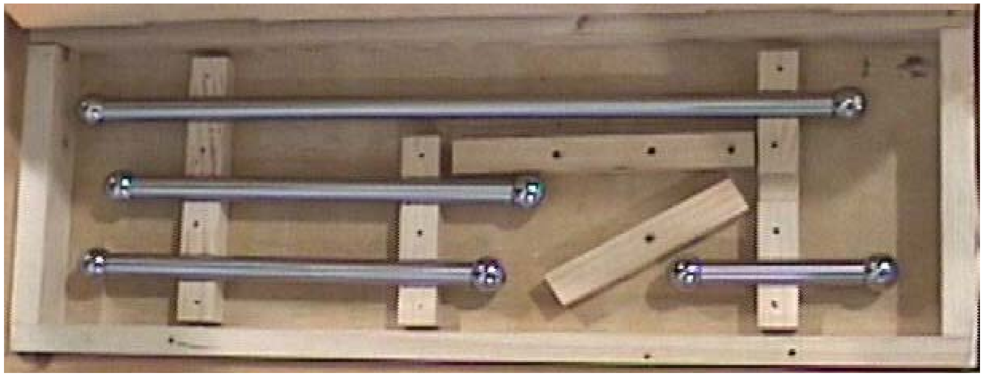
Acceleration Fit Corrections

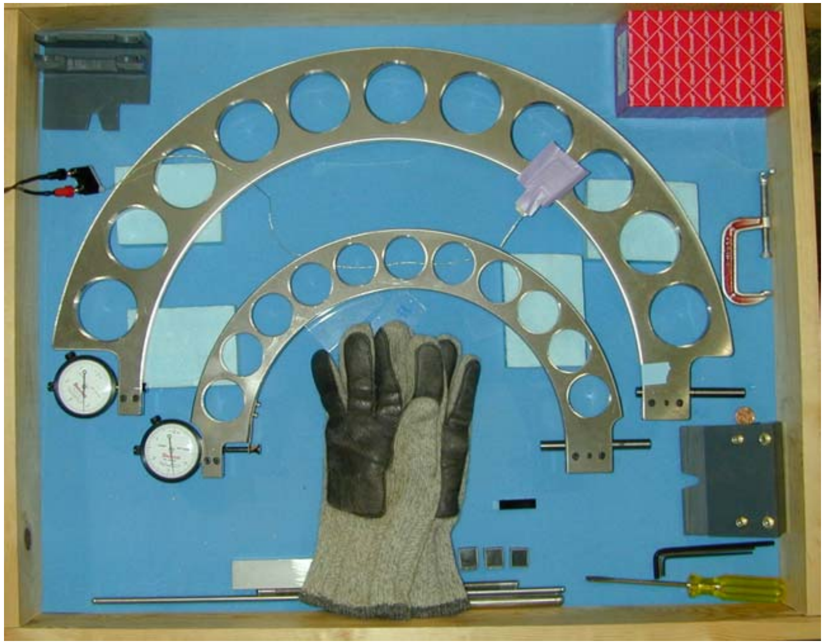
Finite pendulum thickness	1.0007857
Pendulum attachment & imperfections	1.0000433
Data averaging interval (Δt=1s)	1.0000667
Numeric derivative (Δt=10s)	1.0134544
α_{62} correction	0.9998767
α_{82} correction	0.9999951
Magnetic damper torque	1.0000060

Attractor Mass Metrology







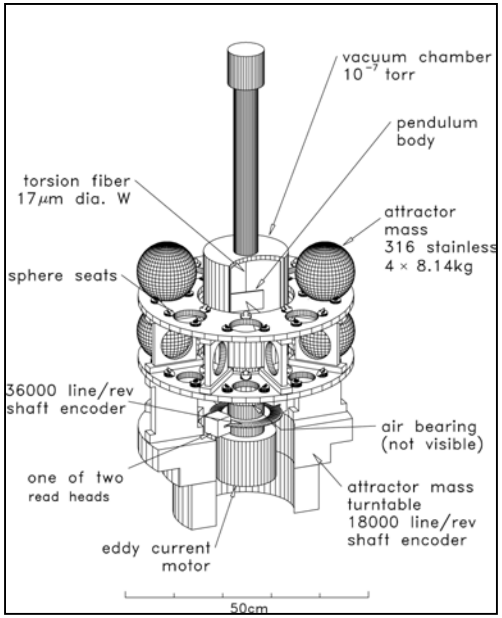


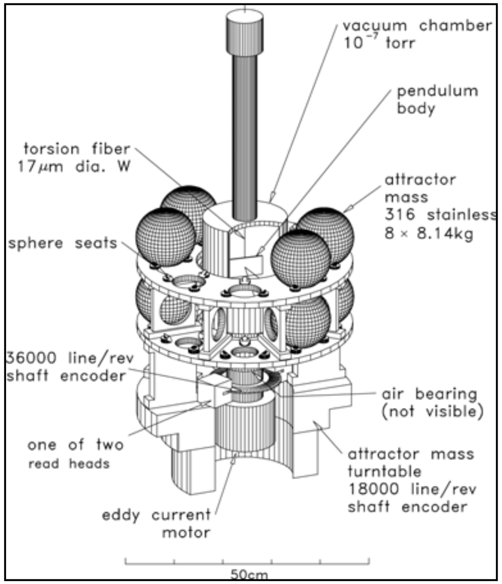
Attention to Details

- Attractor metrology
  - Temperature corrected Q_{22} before & after run showed good reproducibility (6.1 PPM).
  - Q_{22} is temperature corrected with attractor temperature during run.
  - Average over turntable & sphere imperfections using orthogonal configuration pairs.
- Attractor masses
  - Direct equal arm balance measurement ±6mg.
  - The air density (pressure,temperature,…) is recorded during runs and the displaced air mass is subtracted.
  - Take 2 configurations with balls moved 90° on the turntable.
  - Average out non-spherocity & density fluctuations of the balls by rotating.

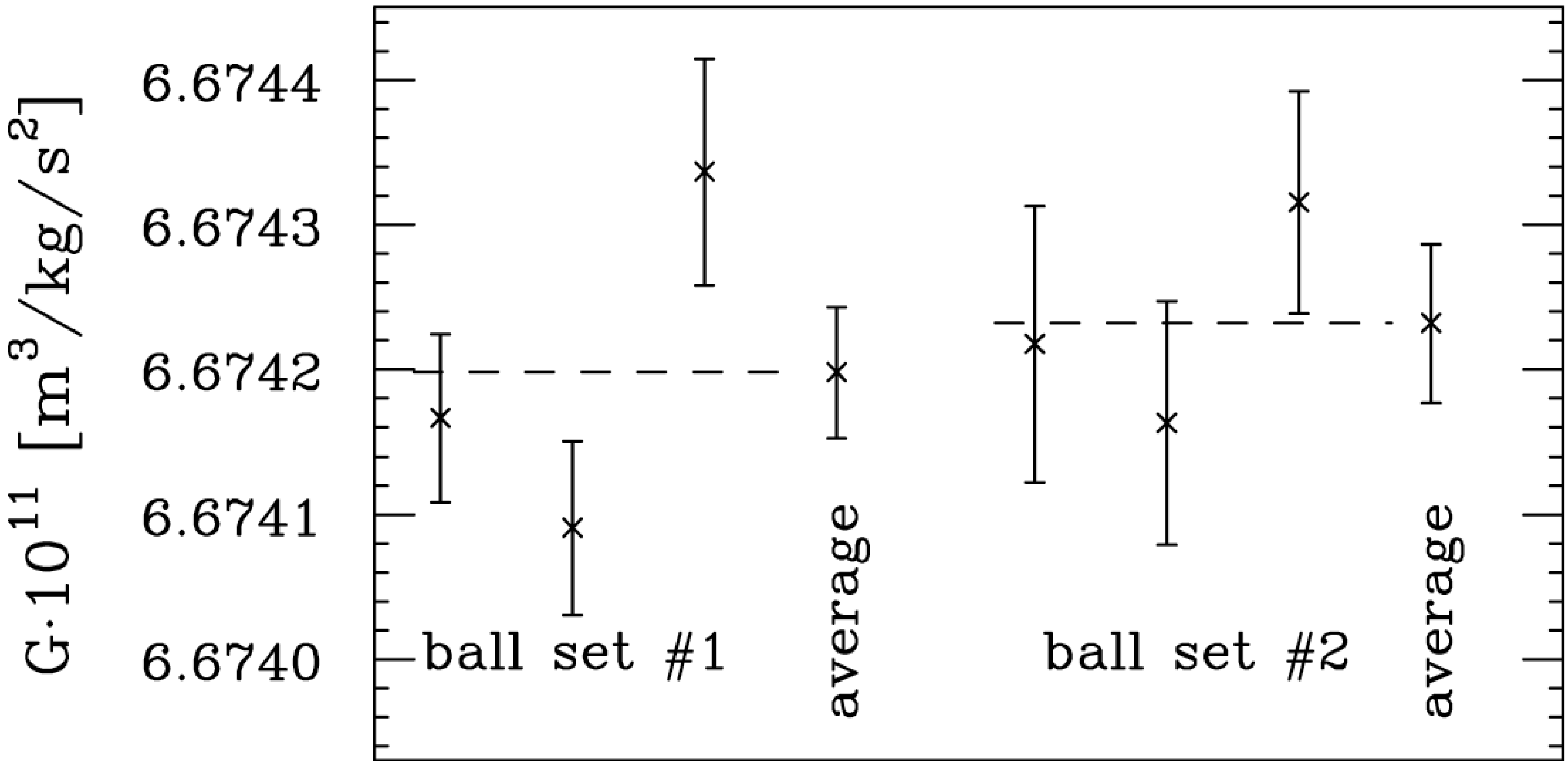
Other Tests

- Various signal speeds
- Various inner turntable speeds
- Large magnetic fields
- Large rotating temperature gradients
- Various sampling times
- Data simulations
- Different sets of 4 spheres
- Linearity with 8 and 4 ball measurements





G Data Sets



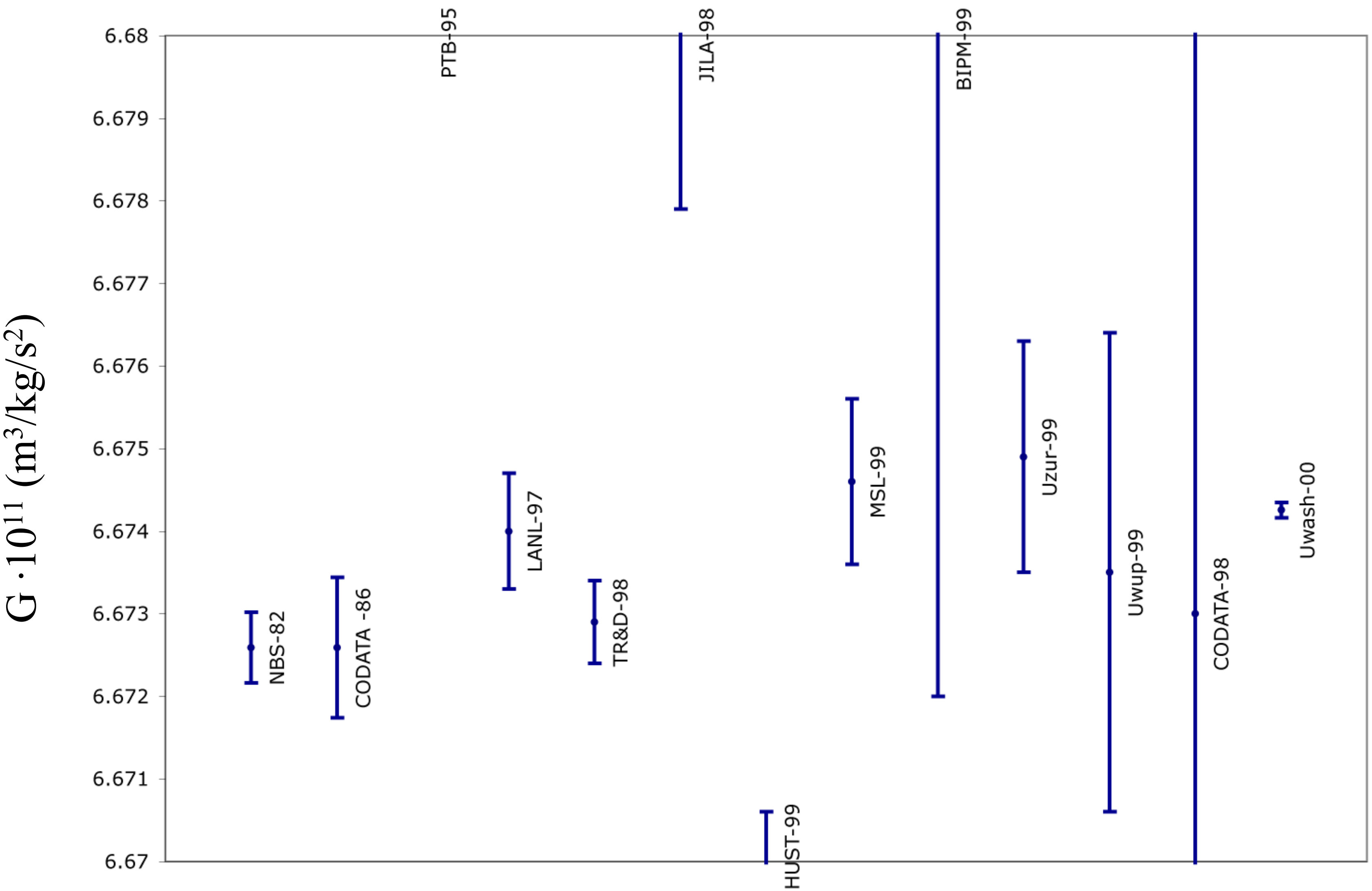
G·10^11 [m^3/kg/s^2]
6.6744
6.6743
6.6742
6.6741
6.6740
ball set #1
average
ball set #2
average

Value for G

$$G = (6.674255 \pm 0.000092) \times 10^{-11}\,\mathrm{Nm}^2/\mathrm{kg}^2$$

Quantity	Measurement uncertainty	ΔG/G (ppm)
Systematic Errors:		
Pendulum:		
Width	20μm	0.4
Thickness & Flatness	4.0μm	4.0
Attractor Masses:		
Diagonal	1.0μm	7.1
Ball-bar Calibration	0.2μm	1.4
Vertical	1.0μm	5.2
Sphere Diameter	1.5μm	2.6
Attractor Temperature	100mK	6.9
Sphere Mass	3.0mg	0.4
Air Humidity		0.5
Remaining Twist:		0.3
Magnetic Field:		0.6
Rot. Temperature Grad.:		0.4
Time Base:	10^{-7}	0.1
Data Reduction:		2.0
Statistical Error		5.8
Total		13.7

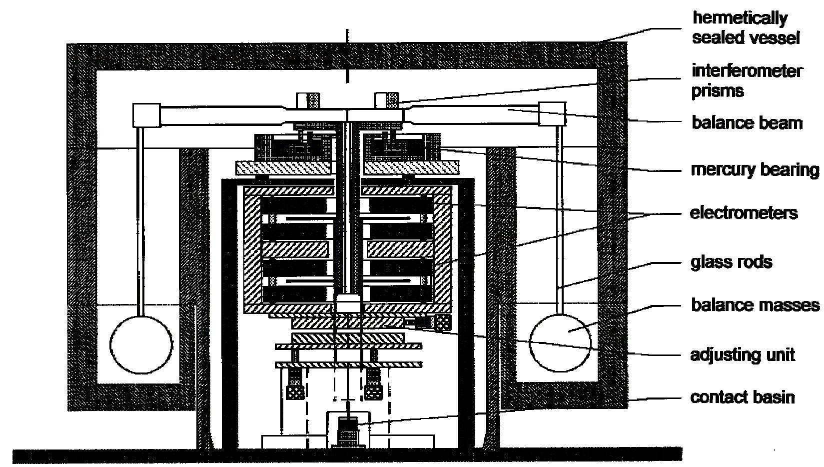
2000 Status

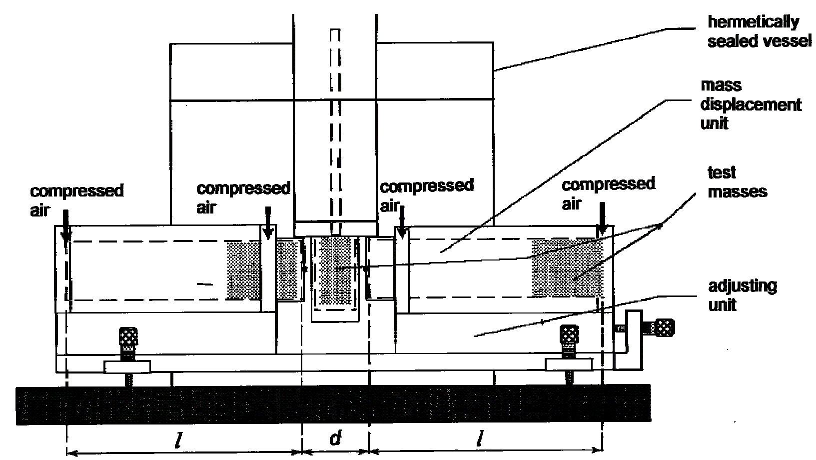


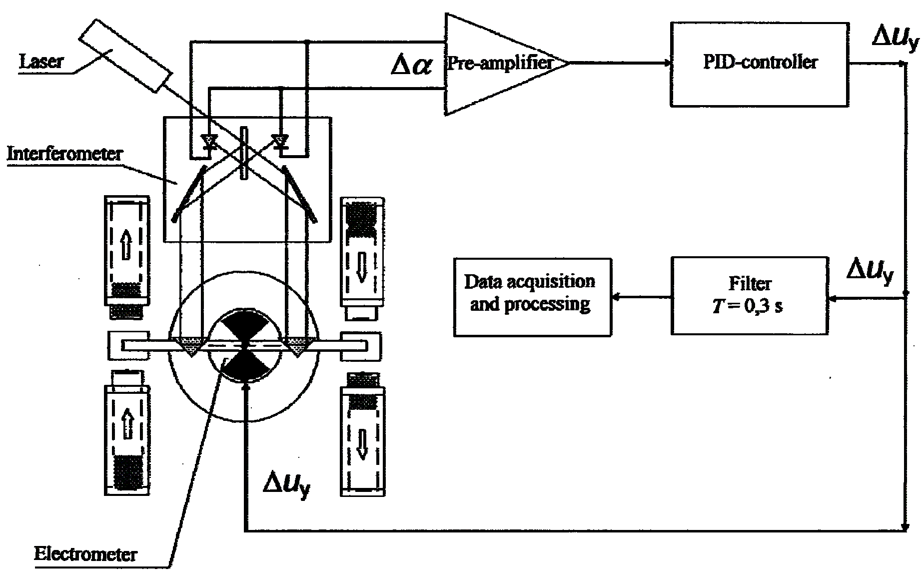
G · 10^11 (m^3/kg/s^2)
6.68
6.679
6.678
6.677
6.676
6.675
6.674
6.673
6.672
6.671
6.67
NBS-82
CODATA -86
PTB-95
LANL-97
TR&D-98
JILA-98
HUST-99
MSL-99
BIPM-99
Uzur-99
Uwup-99
CODATA-98
Uwash-00

PTB Resolution

- Fibreless torsion balance floated in mercury.
- Angular deflection compensated electrostatically.
- AC and DC operation difficult to cross calibrate accuratly.
- Linearity errors between capacitance & angle due to dielectric layers.
- Additional capacitances identified.

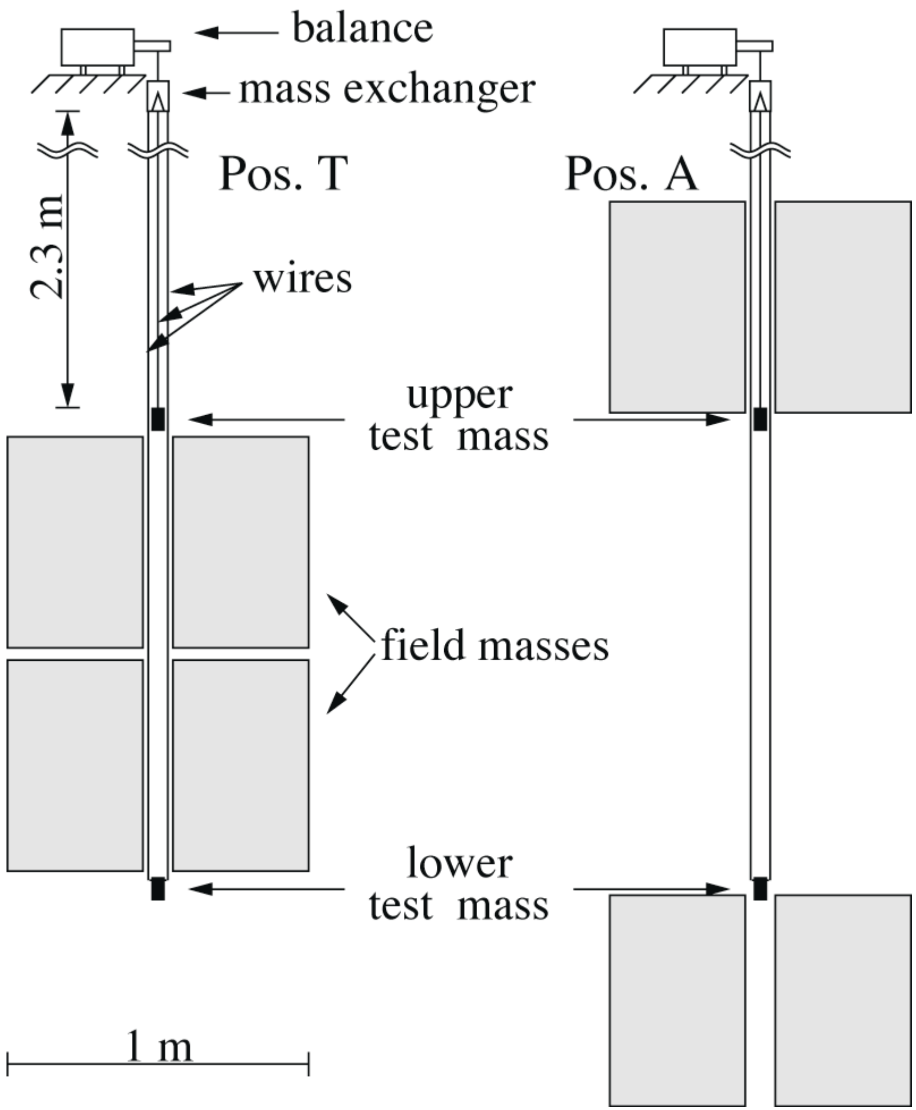


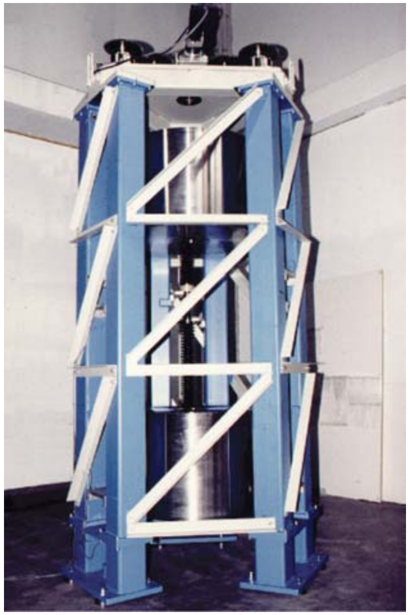




W. Michaelis *et al.*, Metrologia **41**, L29 (2004).
C. C. Speake *et al.*, Phys. Lett. A **263**, 219 (1999).
W. Michaelis *et al.*, Metrologia **32**, 267 (1996).

University of Zurich G Measurement





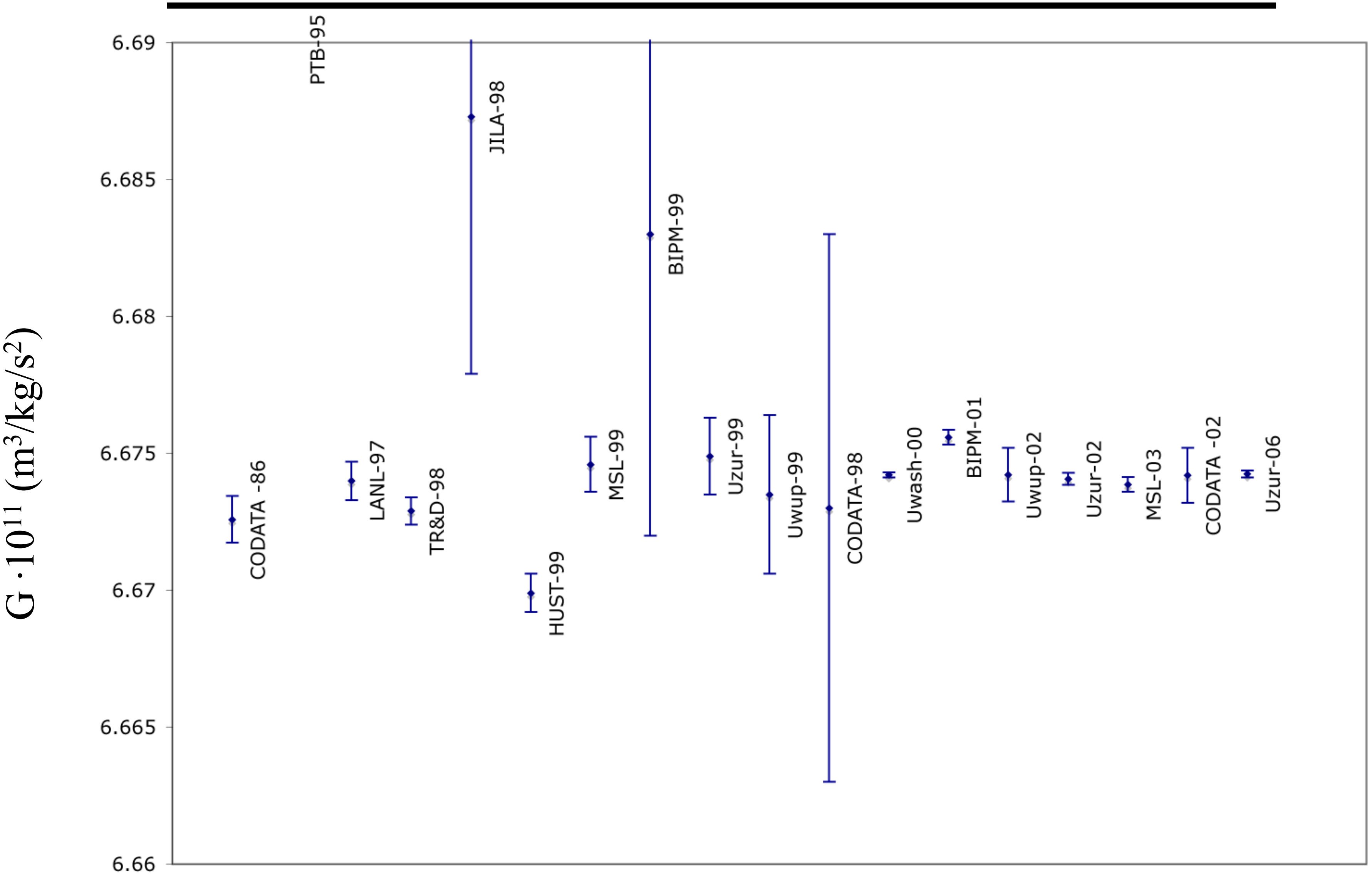
Description	Statistical (ppm)	Systematic (ppm)
Measured signal		
Weighings	11.6	
TM-sorption	7.4	7.4
Linearity	6.1	
Calibration	4.0	0.5
Mass integration	5.0	3.3
Total	16.3	8.1

$$G = (6.674252 \pm 0.000124) \times 10^{-11}\,\mathrm{Nm}^2/\mathrm{kg}^2$$

S. Schlamminger *et al.*, Phys. Rev. Lett. **89**, 161102 (2002)

S. Schlamminger *et al.*, Phys. Rev. D **74**, 082001 (2006)

2002 Value Adjustment



Conclusions

- Big G controversy appears to be resolved.
- Recent measured values are converging.
- Rate of precision improvement appears to be increasing.
- Precision gap with other fundamental constantans narrowing.