Department of Physics, University of Maryland, College Park, MD 20742-4111

Physics 704 **HOMEWORK ASSIGNMENT #5** Fall 2011

Study: P&B 8.1, 8.2, 8.3, 8.6, 8.6.1, 8.7 (not 8.7.1-2), 8.8.2, all of 9 except 9.4

Deadline on May 3.

1. (adapted from van Kampen) Consider a quantized harmonic oscillator interacting with a radiation field. Let n = 0, 1, 2, … be the states of the oscillator, having energies $h\nu\,(n+\frac{1}{2})$.

a) From the dipole selection rule, write down $W(n \mid n')$

and show that $(d/dt)\, p_n = \alpha[(n+1)p_{n+1} - np_n] + \beta[(n-1)p_{n-1} - np_n]$

where the factors α and β depend on the radiation density ρ and frequency ν but not on n.

b) Find the stationary solution p_n^s in terms of (β/α).

c) From equilibrium values p_n^e show that $(\beta/\alpha) = \exp[-h\nu/k_BT]$. (By suitably identifying β and α in terms of ρ, one realizes Einstein's derivation of Planck's law.)

d) Consider a chemical reaction between A and X, with A so abundant that n_A can be taken to be a constant. The forward and reverse reaction rates are k and k´ , respectively. The number n of molecules X has a probability per unit time kn_A to increase by one and $k'n$ to decrease by one. Then we have a similar master equation as in part a):

$(d/dt)\, p_n = k' \,[(n+1)p_{n+1} - np_n] + k\, n_A[p_{n-1} - p_n]$

Show that the stationary solution is a Poisson distribution

2. a) Find the solution to the Fokker-Planck equation (8.67) for the special case A(x) = -x and B(x) = D [or use the first equation [unnumbered] on PB p. 322, an Ornstein-Uhlenbeck process with $A_0=0$ and $A_1=-1$], assuming that $\lim_{t\to 0} P(x,t\,|\,x_0,0) = \delta(x-x_0)$. (Hint: you can find essentially the answer in another context in this chapter.)

b) Suppose in Eq. (8.67) A(x) = k + 1 -x and B(x) = x. Verify that the solution, in the case of a reflecting boundary at x=0 (not discussed in class but see discussion in PB, bottom half of p.325), is [with I_k the modified Bessel function of the first kind]

$$P(x,t\,|\,x_0,0) = \frac{1}{1-e^{-t}}\left(\frac{x}{x_0e^{-t}}\right)^{k/2}\exp\left[-\frac{x+x_0e^{-t}}{1-e^{-t}}\right] I_k\left(\frac{2e^{-t/2}(xx_0)^{1/2}}{1-e^{-t}}\right)$$

3. PB 8.6 (apparently adapted by P&B from van Kampen).

4. Random numbers

a) Consider a linear congruent algorithm $I_{j+1} = aI_j + c \pmod{m}$ for a machine with $m = 2^4 - 1 = 15$.
Taking $a = 7$ and $c = 4$, take a seed $I_0 = 4$ and generate (and list) the first 16 pseudorandom numbers, and find the period.

b) Show that the period depends on the choice of the seed! (E.g. what happens with $I_0 = 11$?)

c) How many repeats do you need to establish that you are in the second period?

5. Use the applet at http://www.math.utah.edu/%7Epa/Random/Random.html to compare the best and the worst random number generators. Here is an easy way to capture graphics: with the Random Graphics window open, enter Alt-PrintScreen, then paste the image into a Word document. The best submission of this problem will be reproduced in the solutions.

P&B problems 9.1 and 9.2 are not assigned, but if you are "conversant" in C, you might take a look at them.