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Physics 704 **HOMEWORK ASSIGNMENT #4** Fall 2011

Study: P&B 6.6, 7.4-7.6
Skim: P&B 7.7

Due date for problems on April 7 [deadline on April 12].

A useful result is that the singular part of the free energy $\propto \xi^{-d}$.

1. Using the variables $x = \exp(-4\beta J)$ and $\mathscr{h} = \beta h$, show that for small x and h the free energy and magnetization of the 1D Ising model given in Eqs. (3.37) and (3.38) can be recast in scaling form. What are the values of the two y's? [This is problem 8.4 of Yeomans' text. She notes earlier that $G/N \to -J - h$ as $T \to 0$; it is the second term that gets recast.]

2. The non-linear σ model describes unit spins with n components. In 2D the recursion relations for temperature T and magnetic field h are (with $b = \exp(\ell)$)

$$dT/d\ell = [(n-2)/2\pi]\, T^2 \qquad\qquad dh/d\ell = 2h$$

a) Show that as $T \to 0$, the correlation length diverges like

$$\xi(T,h) = \exp[2\pi/(n-2)T]\;\; g_1\big(h \exp[4\pi/(n-2)T]\big)$$

where g_1 is some unspecified scaling function. Hints: Integrate the two equations to find $h(\ell)$ and $T^{-1}(\ell)$. Use the standard scaling relation to relate $\xi(T,h)$ and $\xi(T(\ell),h(\ell))$. Starting from T and h close to 0, renormalize until $T(\ell^*) \sim 1$.

b) Write down the singular form of the free energy as $T, h \to 0$.

c) How does the χ susceptibility diverge as $T \to 0$ for $h = 0$?

3. When T approaches the transition temperature T_{KT} from above, the correlation length In the Kosterlitz-Thouless model exhibits an essential singularity $\xi(t) \sim \exp(B/t^{1/2})$, where $t = (T - T_{KT})/T_{KT}$. [See e.g. J M Kosterlitz , "The critical properties of the two-dimensional xy model", J. Phys. C 7, 1046 (1974).] (For $t<0$, ξ is 0.)

a) Use finite-size scaling theory to gauge the shift in T_{KT} for systems with finite size L.

b) Show that, for a system such as the XY model, the specific heat does not diverge, and in fact has no observable singular behavior, at the Kosterlitz-Thouless transition at T_{KT}.

4. a) In the Kosterlitz-Thouless model, draw a vortex configuration for S = +2 and another for S = –2, where S (often called n) is the "charge" of the vertex.

b) Confirm that the solution of the Kosterlitz equation $(dx/d\ell) = a^2y^2 = x^2 + ct$ is

$$\ell = \ell_0 + (ct)^{-1/2} \arctan[x(ct)^{-1/2}]$$