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Physics 704 **HOMEWORK ASSIGNMENT #3** Fall 2011

Study: P&B 3.6,6.3-6.7,7.1-7.2
Skim: P&B 6.2

Due date for problems on March 15 [deadline on March 17].

1. P&B problem 3.8

2. Consider the solid-on-solid (SOS) model of an interface. In each column of a 1D lattice i, the interface lies at a position n_i = 0, 1, 2, 3, ..., which is single-valued (so no overhangs or voids):

$$\mathcal{H} = \varepsilon \Sigma_i |n_i - n_{i+1}| - K \Sigma_i \delta_{n_i,0} \qquad n_i = 0, 1, 2, 3, \ldots$$

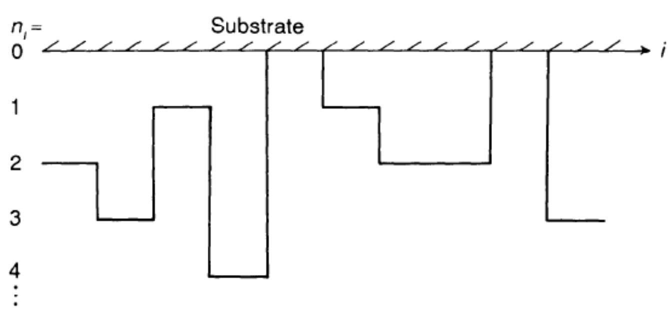

Fig. 5.2. The solid-on-solid model of an interface on a semi-infinite lattice. In each column of the lattice, i, the interface lies at a position $n_i \geq 0$.

I have no idea why Yeomans drew this picture with increasing n_i going down; I would have drawn it upside down, with n_i increasing on the ordinate.

a) Write down the transfer matrix of this model in terms of $x = \exp(-\varepsilon / k_B T)$ and $\kappa = \exp(K/k_B T)$. (Do not symmetrize the K term.)

b) By considering an eigenvector of the form

$(\varphi_0, \cos(q+\theta), \cos(2q+\theta), \ldots)$

show that there is a continuous spectrum of eigenvalues

$(1-x)/(1+x) \leq \lambda \leq (1+x)/(1-x)$.

Hint: To do the sums that arise, it is easier to write $\cos(mq+\theta)$ as $\mathrm{Re}[\exp[i(mq+\theta)]]$. Alternatively and more simply, you can use the form in the next part and then recognize that μ is not real but instead pure imaginary.

c) Show that for $\kappa > (1-x)^{-1}$, there is also a bound state eigenvector of the form

$(\varphi_1, e^{-\mu}, e^{-2\mu},\ldots)$ corresponding to the eigenvalue $\lambda_1 = [\kappa(1-x^2)(\kappa-1)]/[\kappa(1-x^2)-1]$.

One can also show that this λ_1, when it exists, is the largest eigenvalue; thus, it dominates the thermodynamics. Hence the critical value of K is given via $\kappa_c = (1-x)^{-1}$. (Adapted from Yeomans)

3. P&B problem 6.5

Hints for part a): The classical values of these critical exponents are $\nu_H = 1/3$ and $\eta_H = 0$. Start with the form of the Landau-Ginzburg equation on the critical isotherm:

$$h(\mathbf{r}) = g_3(T_c)m^3(\mathbf{r}) + g_4(T_c)m^5(\mathbf{r}) + \ldots - A\nabla^2 m(\mathbf{r})$$

Let $h(\mathbf{r}) = h_0 + h_1\,\delta(\mathbf{r})$ and $m(\mathbf{r}) = m_0(T_c,h) + \varphi(r)$ and keep only the leading power of m_0 and terms linear in φ. More hint: in 3D, $\varphi(r) = [(h_1/2\pi A)/r]\exp(-r/\xi)$.

4. Consider the q-color version of the 1D q-state Potts model on a large N-spin ring:

$$H = -J\sum_{i=1}^{N} \delta_{s_i,s_{i+1}}, \text{ where } s_{N+1} \equiv s_1$$

a) Write down the transfer matrix and consider its simple form. One eigenvalue, the largest (λ_1), corresponds to an eigenvector for which all components are equal. Find its value. The remaining $q-1$ eigenvalues are degenerate; show that each is $\lambda_1 - q$.

b) Calculate the free energy per site in the thermodynamic limit.

c) Using the eigenvalues from part a), write down the correlation length ξ and discuss its behavior as $T \to 0$.

d) i) Show that this system is critical ($\xi \to \infty$) at $T = 0$.
ii) In the antiferromagnetic case ($J < 0$), is the system still critical at $T = 0$?