Physics 603: Midterm Test Name (*print*): ___________________________________

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

March 13, 2012 ***Sign* Honor Pledge:____**

1. Consider N atoms, each of which can be either in its ground state or an excited state of energy Δ. Suppose that N_1 atoms are in the excited state (and so $N_0 = N - N_1$ in the ground state), so that the energy $E = N_1\Delta$.

a) Find the number of configurations giving N_1 excited atoms and use the Stirling0 approximation to show that the entropy is

$$S(E,N) = -Nk_B\left[\left(\frac{E}{N\Delta}\right)\ln\left(\frac{E}{N\Delta}\right)+\left(1-\frac{E}{N\Delta}\right)\ln\left(1-\frac{E}{N\Delta}\right)\right]$$

b) Is this entropy extensive? Justify your answer.

c) Identifying E as the internal energy U, i) find the temperature T of this system. ii) Then invert your answer to get U(T).

d) Find the heat capacity of this system.

e) Compare your results with those for spin-1/2 dipoles in a magnetic field, as treated in class and in PB §3.10.

2. Suppose $\Phi(x) = x^3$ and $X = d\Phi/dx$. Perform a Legendre transformation to find $\Psi(X)$ for $x \geq 0$.

3. Consider a semi-infinite 1D system for a single classical particle moving vertically ($z \geq 0$)

with Hamiltonian $\mathcal{H}(p_z, z) = p_z^2/2m + A z^n$, $n>0$

a) Write down the canonical distribution function $\rho(p_z, z)$ and show that it separates into the product of two factors, one dependent only on p_z and the other only on z.

b) Find the mean potential energy $\langle A z^n\rangle$ ($z \geq 0$) of the particle ($z \geq 0$), and compare your result with the generalized equipartition derived in class for modes $\propto$ even powers of z for $-\infty < z < \infty$.

Reminder: $\int_0^\infty \exp\left(-x^n\right)dx = \Gamma\left(1+\frac{1}{n}\right)$, $n>0$

In-exam hint/reminder: if one knows $\int \exp[-a\,f(x)]\,dx$ one can find $\int f(x)\exp[-a\,f(x)]\,dx$ using the trick shown in class (hint: derivative).

4. [Adapted from a qualifier problem] Consider a low-density ideal gas of N atoms confined to a container of volume V and internal surface area A. While the interactions between atoms can be neglected, there is an attraction between atoms and the walls which must be taken into account. A simple model for the N′ atoms that are adsorbed onto the surface is to treat them as a two-dimensional (2D) classical ideal gas, where the energy of an adsorbed atom with 2D momentum **p** is

$$\varepsilon(\mathbf{p}) = |\mathbf{p}|^2/2m - \varepsilon_0 \qquad \text{(Treat } \varepsilon_0 \text{ as some given positive binding energy.)}$$

a) Show that the Helmholtz free energy of the N′ [indistinguishable] adsorbed atoms bound to the surface is

$$F(T,A) = -\beta^{-1}\,[-N' \ln N' + N' + N' \ln A - 2\,N' \ln \lambda_T + N'\,\beta\,\varepsilon_0]$$

b) What is the chemical potential μ_s of the adsorbed atoms?

c) From your notes or P&B, recall that the chemical potential μ of the N – N′ atoms in the container is

$$\mu = k_B T \ln\,[(N-N')\lambda_T^3/V]$$

When the atoms in the volume and those on the surface are in equilibrium with each other, what is the average number of atoms adsorbed as a function of temperature T?

d) Find the fraction of atoms sticking to the walls in the limits $T \to 0$ and $T \to \infty$, and briefly explain why these limits are reasonable.