Department of Physics, University of Maryland
College Park, MD 20742-4111

Physics 603 **HOMEWORK ASSIGNMENT #1** Spring 2014

Due date for problems on Thursday, Feb. 6 [deadline on Feb. 11].

1. a) Find the Legendre transform $\Psi(P)$ of $\Gamma(X) = X^3$.

b) Carry out an explicit Legendre transformation for a more complicated function: Consider the thermodynamic potential Y:

$$Y(U,X) = A + BU + CX^2 + DU^2 + EU^2X^2$$

Calculate $W = (\partial Y/\partial U)_X$ and $P = (\partial Y/\partial X)_U$

Construct explicitly the thermodynamic potential $\Psi(W,X)$ and from it verify the relations

$$U = -(\partial\Psi/\partial W)_X \quad \text{and} \quad P = (\partial\Psi/\partial X)_W$$

2. a) Derive the Maxwell relation associated with H(S,p).

b) Verify the Maxwell relation $(\partial S/\partial V)_T = (\partial p/\partial T)_V$. From which thermodynamic function does it originate?

c) Extend U to $U(S,V,m)$ and G to $G(T,p,m)$ by adding $H\,\mathrm{d}m$ (i.e. the magnetic work ON an object is $H\,\mathrm{d}m$, analogous to $-p\,\mathrm{d}V$ for mechanical work), and write down the new Maxwell relations involving H and/or m that result. (Note that this formulation includes the magnetic energy of the object; cf. the Kittel posting. Here m is taken as extensive, because M curiously is conventionally reserved for magnetization density (i.e., magnetization per volume). In older books, the extensive magnetization is often written $\mathfrak{M}$ [actually in Germanic Faktur, but Old English is the closest on my computer].)

3. (essentially Kardar 1-7) For an elastic filament it is found that, at a finite range in temperature, a displacement x requires a force

$$J = ax - bT + cTx \quad ,$$

where a, b, and c are constants. Furthermore, its heat capacity at constant displacement is proportional to temperature: $C_x\,[= T\,\partial S/\partial T|_x] = A(x)\,T$.

a) Use an appropriate Maxwell relation to calculate $\partial S/\partial x|_T$.

b) Show that A must be independent of x: i.e., $\mathrm{d}A/\mathrm{d}x = 0$.

c) Calculate $S(T,x)$, assuming $S(0,0) = S_0$.

d) Show that the heat capacity at constant tension, $C_J = T\,\partial S/\partial T|_J$, can be written as

$$C_J = T\left[A + \frac{(ab - cJ)^2}{(a + cT)^3}\right]$$

4.

Problem I.3

Equilibrium is a central concept in Statistical Physics.

(a) ~~[6 points]~~ Give a definition of equilibrium for

  (a) an isolated system

  (b) a pair of interacting systems

(b) ~~[6 points]~~

  (a) For a closed isolated system, give two examples of experimentally measurable thermodynamic variables, their definitions, and what equilibrium implies about them.

  (b) For a system that can exchange particles, give one more example of an experimentally measurable thermodynamic variable and its definition.

(c) ~~[7 points]~~ A colleague gives you a crystalline solid sample and asks you to examine its electronic properties (e.g. voltage V versus current I).

  (a) What are the thermodynamic variables and potentials that are relevant to your measurement?

  (b) As an experimentalist, why should you concern yourself with the sample equilibrium?

(d) ~~[6 points]~~ A laboratory is never in equilibrium, in the sense of an isolated system. Give four examples of likely nonequilibrium conditions for a scientific laboratory in this building (or another campus building if you prefer).