Formulas for Final, PHYS404, Fall 2012

$$\Omega(N, N_\uparrow) = \frac{N!}{N_\uparrow!\left(N - N_\uparrow\right)!}$$

$$\Omega(N, q) = \frac{(q+N-1)!}{q!\left(N-1\right)!}$$

$$\Omega(U, V, N) = f(N) V^N U^{fN/2}$$

$U = (f/2)\, N k_B T$; determining f: 3 for atoms,
5 for diatomic molecules at low T, 7 at higher T,
2 for each direction of an Einstein oscillator

Ideal gas law $pV = Nk_B T = nRT$

Entropy $S = k_B \ln \Omega = \int dQ_{rev}/T$ $\quad 1/T = (\partial S/\partial U)_{V,N}$
Ideal gas $S = Nk_B\,[(V/(N\Lambda^3) + 5/2]$; $\Lambda_T = h/\sqrt{(2\pi m k_B T)}$
Paramag $S = Nk_B\,[\ln(2\cosh x) - x\tanh x]$; $x = \mu B/k_B T$
$U = -\mu B \tanh(\beta\mu B)$; $\quad M = N\mu\tanh(\beta\mu B)$

$\Delta U = Q + W_{on} = Q - W_{by}$ $\qquad \Delta S \geq 0$

Constants: $k_B \approx 10^{-4}$ eV/K = 1.38×10^{-23} J/K
C_V of 1 gm of water (ice) is 1 cal/K (~½ cal/K)
R ~ 8.3J/K $\quad N_A = 6.02\times10^{23}$
Stirling: $\ln n! \approx n \ln n - n$

Expansions in $\varepsilon \ll 1$:
$\ln(1\pm\varepsilon) \approx \pm\varepsilon\,[-\varepsilon^2/2]$, $\quad \exp(\pm\varepsilon) \approx 1\pm\varepsilon\,[+\varepsilon^2/2!]$
$\Gamma(n+1) = n!$ (integer n); $\Gamma(1/2) = \sqrt{\pi}$; $\Gamma(x+1) = x\Gamma(x)$

Change in internal energy, change in temperature, heat, work, during "simple" processes:
isobaric ($\Delta p = 0$), isochoric ($\Delta V = 0 = W$), isothermal ($\Delta U = \Delta T = 0$), adiabatic ($Q = 0$).
Along an isobar, $W_{by} = p(V_f - V_i)$; along an isotherm $W_{by} = Nk_B T \ln(V_f/V_i)$
Along an adiabat pV^γ is constant, as is (using the ideal gas law) $TV^{\gamma-1}$ Note $\gamma = (f+2)/f$

Helmholtz and Gibbs free energies $\quad F(T,V,[N]) = U - TS + \mu N$ $\qquad G(T,p,[N]) = U + pV - TS + \mu N$
$U(V,S\,[,\mu])$ $\quad H(p,S\,[,\mu])$ $\qquad \Phi = U - TS - \mu N$

Thermo. identities: $dU = TdS - p\,dV + \mu dN$
$dF = -S\,dT - p\,dV + \mu\,dN$
$dG = -S\,dT + V\,dp + \mu\,dN$, etc.

Maxwell relations: 2nd deriv of thermo functions do not depend on order of derivs

$\Delta S_{mix} = -Nk_B[x\ln x + (1-x)\ln(1-x)]$

$\beta = V^{-1}\,\partial V/\partial T|_p \qquad \kappa_T = -V^{-1}\,\partial V/\partial p|_T$
$S = \int C/T\, dT$
$F = -k_B T \ln Z \qquad U = -\partial \ln Z/\partial\beta$
$Z_N = Z_1^N/N!$ for indistinguishable

Non-relativistic gas $Z_1 = (V/\Lambda_T^3)\, Z_{int}$

$Z = \Sigma_i\, g_i\, e^{-\beta\varepsilon i}$ $\qquad \mathcal{Z} = \Sigma_i\, g_i\, e^{-\beta(\varepsilon i - \mu Ni)} = \Sigma_N\, z^N Z_N$
[NB εi means ε_i]

$Z_{rot} \approx k_B T/2\varepsilon$ for $k_B T \gg \varepsilon$, dimer of identical atoms

Maxwell 3D $\quad D(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2k_B T}\right)$

Ideal gas $\Phi = -k_B T \ln \mathcal{Z} = -pV$

$\mathcal{Z}_{FD/BE} = (1 \pm \exp[-\beta(\varepsilon-\mu)])^{\pm1}$ $\qquad \beta = 1/k_B T$

$$\bar{n}_{\frac{FD}{BE}}(\varepsilon; T, \mu) = \frac{1}{\exp[\beta(\varepsilon-\mu)]\pm1} = \frac{1}{z^{-1}\exp(\beta\varepsilon)\pm1}$$

$$\varepsilon \propto n^p: \; \varepsilon = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) = \frac{h^2 n^2}{8mL^2} \;\text{ or }\; \varepsilon = \frac{hcn}{2L}$$

$\mathcal{G}_{FD}(\varepsilon) \propto (\varepsilon/\varepsilon_F)^{(D/2)-1}/\varepsilon_F$; $\mathcal{G}(\varepsilon) \propto \varepsilon^{(D/p)-1}$

$$\int^\infty h(\varepsilon)\bar{n}_{FD}(\varepsilon;T)\,d\varepsilon = \int_0^{\varepsilon_F} h(\varepsilon)\,d\varepsilon + \frac{\pi^2}{6}\left(k_B T\right)^2\left[h'(\varepsilon_F) - h(\varepsilon_F)\frac{G'(\varepsilon_F)}{G(\varepsilon_F)}\right]$$ [NB: G is $\mathcal{G}$]

Photons $u(\varepsilon) = [8\pi\varepsilon^3/(hc)^3]\ \bar{n}_{BE}(\varepsilon; T, 0)$ $\qquad \varepsilon = hf$

Wien: $\lambda_{max}T = 0.0029$ m K

Power from perfect blackbody radiator: $\sigma\times\text{area}\times T^4$

Debye $\quad T_D = (hc_s/2k_B)(6N/\pi V)^{1/3}$

$$U = \frac{9Nk_B T^4}{T_D^3}\int_0^{T_D/T}\frac{x^3}{e^x - 1}dx$$

BEC $\quad \Gamma(\nu)\, Li_\nu = \int_0^\infty \frac{x^{\nu-1}dx}{z^{-1}e^x - 1}$ $\qquad N = (V/\Lambda_T^3) Li_{3/2}(z)$

$U = (3/2)\, k_B T\, Li_{5/2}(z)$

$E_{Ising} = -J\,\Sigma_{\langle i,j\rangle}\, s_i\, s_j$