

**Due date:** Tuesday, May 12      **Deadline:** Thursday, May 14, 2pm

B means a problem in Blundell & Blundell's text; GT means a problem in Gould & Tobochnik.

1. B29.2, a short problem
2. Consider a Fermi gas in 2 rather than 3 dimensions. (Such 2D electron gases can be found on the surface of some semiconductors, for example.)
  - a) Find the Fermi energy in terms of number of fermions  $N$  and area  $A$
  - b) Show that the density of states is independent of  $\epsilon$  and write it in terms of  $\epsilon_F$  and  $N$ .
  - c) Show that the average energy of the fermions is  $\epsilon_F/2$ .
  - d) One can show (but you do not have to) that  $\mu(T) = k_B T \ln(e^{\epsilon_F/k_B T} - 1)$ .

Find the limits of  $\mu(T)$  as i)  $T$  goes to 0 and ii) when  $k_B T \gg \epsilon_F$  and iii) show that they make sense.

- e) One can also show (but you do not have to) that for low temperature  $U/N - \epsilon_F/2 = (\pi^2/6)(k_B T)^2/\epsilon_F$ .

Find the associated specific heat and compare to the result in 3D.

3. B30.5 Note that  $\text{Li}_1(z) \rightarrow \infty$  as  $z \rightarrow 1$ . (In Mathematica,  $\text{Li}_1(z)$  is  $\text{PolyLog}[1, z]$ .) The problem should take you only a few lines to do.

NOT assigned B29.6, which is long and involved, but I will provide solutions. Note that in eqs. (29.31) and (29.32),  $n_j$  should be  $n_j!$ , so that the denominators in both cases are  $n_j! (g_j - n_j)!$ .

Furthermore, in eq. (29.35), there is this same error along with another in the term with parentheses: the denominator should be  $n_j! (g_j - 1)!$ . In going from eq. (29.35) to (29.36), you should assume that  $g_j \gg 1$  so that you can replace  $g_j - 1$  by  $g_j$ .

The expression to be maximized is

$$\frac{S}{k_B} + \alpha \left( N - \sum_j g_j \bar{n}_j \right) + \beta \left( E - \sum_j g_j \bar{n}_j E_j \right)$$

This is, I believe the source of using  $\beta$  for  $1/k_B T$ .