Problem Set ---Due February 17

1) Consider a system whose angular state in the standard $|lm\rangle$ basis is given by $|\psi\rangle = \sqrt{\tfrac{1}{2}}\left(|22\rangle + e^{i\gamma}|21\rangle\right)$ where γ is a phase. Show that $\langle\psi|\hat{L}_x|\psi\rangle = \cos(\gamma)$. The fact that the expectation value depends on the phase γ is a demonstration of the general principle that while the overall phase of a states is unphysical, the relative phase between different components has physical significance.

2) Consider a system with a central potential. As noted in class the energy eigenstates are of the form $|n\,l\,m\rangle$ where *n* is the radial quantum number and *l* and *m* describe the angular degrees of freedom. Suppose the system is in the state $|\psi\rangle = \frac{1}{2}\left(|100\rangle - |200\rangle - \sqrt{\tfrac{1}{2}}|210\rangle + i\sqrt{\tfrac{1}{2}}|211\rangle + \sqrt{\tfrac{1}{10}}|221\rangle - \tfrac{3}{\sqrt{10}}|22-1\rangle\right)$.Verify that the state is normalized and calculate the following expectation values:
$\langle\psi|\hat{L}_z|\psi\rangle$
$\langle\psi|\hat{L}^2|\psi\rangle$
$\langle\psi|\hat{L}_z^2|\psi\rangle$
$\langle\psi|\hat{L}_y|\psi\rangle$
$\langle\psi|\hat{L}_x|\psi\rangle$

3) In class we showed how to construct explicit matrices for the operators $\hat{L}_x, \hat{L}_y, \hat{L}_z$ and $\hat{L}^2$ and that the matrices were block diagonal.
   a) Working in the block with *l*=1 explicitly construct the 3 by 3 matrices associated with these four operators.
   b) Denote the matrices associated with $\hat{L}_x, \hat{L}_y, \hat{L}_z$ as $\vec{L}_x, \vec{L}_y, \vec{L}_z$ show that they satisfy the same commutation relations as the operators:
   $\left[\vec{L}_x, \vec{L}_y\right] = i\hbar\vec{L}_z$
   $\left[\vec{L}_y, \vec{L}_z\right] = i\hbar\vec{L}_x$
   $\left[\vec{L}_z, \vec{L}_x\right] = i\hbar\vec{L}_y$
   c) Show that as matrices $\vec{L}_x^{\,2} + \vec{L}_x^{\,2} + \vec{L}_x^{\,2} = \vec{L}^2$ where $\vec{L}^2$ is the matrix associated with $\hat{L}^2$.

4) We arbitrarily choose to work with a basis of states which were eigenstates of $\hat{L}_z$ and $\hat{L}^2$ we equally well could have picked $\hat{L}_x$ and $\hat{L}^2$. Let us denote the new basis as the primed basis. The primed basis states are normalized and have the following properties
$\hat{L}_x|l\,m\rangle' = \hbar m|l\,m\rangle'$ (i)

$\hat{L}^2 |l\, m\rangle' = \hbar^2\, l(l+1) |l\, m\rangle'$ (ii)

by direct analogy with the usual basis states. Since each basis set is complete any of the primed basis states can be written as a superposition of unprimed states with the same l: $|l\, m_1\rangle' = \sum_n c_{l\, m_2\, m_1} |l\, m_2\rangle$ where $c_{l\, m_2\, m_1}$ are coefficients.

a) Using the properties (i) and (ii) above the matrix elements we computed in class plus normalization to show that

$|11\rangle' = \frac{1}{2}|11\rangle + \sqrt{\frac{1}{2}}|10\rangle + \frac{1}{2}|1-1\rangle$

$|10\rangle' = \sqrt{\frac{1}{2}}|11\rangle + \sqrt{\frac{1}{2}}|1-1\rangle$

$|1-1\rangle' = \frac{1}{2}|11\rangle - \sqrt{\frac{1}{2}}|10\rangle + \frac{1}{2}|1-1\rangle$

b) Explain how the linear combinations given in part a) can be understood in terms of the eigenvectors and eigenvalues of the matrix $\vec{L}_x$ constructed in problem 3.