Problem Set ---Due February 10

1) In class we discussed the separation of variables for the three dimensional time-independent Schrödinger equation with a spherically symmetric potential. In this problem I want you to work thorough the equivalent for a two dimension system with an axially symmetric potential. In particular consider the equation $\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi + V(r)\psi = E\psi$ in polar coordinates. Show that the it has solutions of the form $\psi(r,\vartheta) = R(r)\Theta(\vartheta)$ with $\Theta(\vartheta) = e^{im\vartheta}$ (integer *m*) and *R* satisfying $\frac{-\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\right)R(r) + \left(V(r)+\frac{m^2\hbar^2}{2mr^2}\right)R(r) = E\,R(r)$.

2) Consider the infinite spherical square well of radius *a* studied in class. We found the energies of the *l=0* states but showed that the *l*=1 states could only be solved numerically by finding the roots of a transcendental equation. Find a numerical estimate for the energies of the lowest three by solving this equation numerically.

3) The spherical harmonics are a complete orthonormal set describing smooth angular functions. Completeness means than any smooth angular function may be written in the form $f(\vartheta,\phi) = \sum_{l,m} c_{l,m}\, Y_l^m(\vartheta,\phi)$. Orthonormailty implies $\sum_{l,m} |c_{l,m}|^2 = 1$. Find the coefficients $c_{l,m}$ for the function $f(\vartheta,\phi) = (\sin(\vartheta)+\sin(2\vartheta))\cos(\phi)$.

4) In class it was claimed that the spherical harmonics were orthonormal in the sense that $\int_0^{2\pi} d\phi \int_0^{\pi} d\vartheta \sin(\vartheta)\left(Y_{l'}^{m'}(\vartheta,\phi)\right)^* Y_l^m(\vartheta,\phi) = \delta_{l.l'}\delta_{m.m'}$ .Explicitly show that this holds for the subset $Y_2^2(\vartheta,\phi)$, $Y_2^1(\vartheta,\phi)$ and $Y_2^0(\vartheta,\phi)$ and $Y_1^0(\vartheta,\phi)$ (where the explicit forms of these are as given in the book or class) by directly do the intergrals.

5) Find the following matrix elements for states labeled $|l,m\rangle$;
   a) $\langle 1,2|L_z|2,2\rangle$
   b) $\langle 2,2|L_x|2,2\rangle$
   c) $\langle 2,2|L_x|2,1\rangle$
   d) $\langle 2,2|L_x|1,1\rangle$
   e) $\langle 2,2|L_x^2|2,0\rangle$
   f) $\langle 2,2|L_x^2 + L_y^2|2,2\rangle$
   g) $\langle 2,1|(L_x^2 + L_y^2)L_z|2,1\rangle$