Due February 3rd

1. Consider the construction used in class to combine two Hilbert spaces with the basis states in the combined space of the form $|i,j\rangle = |i\rangle^{(1)} |j\rangle^{(2)}$ and operators of the form $\hat{C} = \hat{A}^{(1)}\hat{B}^{(2)}$. The operation of $\hat{C}$ on the basis states in given by $\hat{C}|i,j\rangle = \hat{A}|i\rangle^{(1)}\,\hat{B}|j\rangle^{(2)}$. Show that if $\hat{A}^{(1)}$ and $\hat{B}^{(2)}$ are Hermitian in their respective subspaces then $\hat{C}$ is Hermitian in the combined space.

2. Consider the construction used in class to combine two Hilbert spaces with the basis states in the combined space of the form $|i,j\rangle = |i\rangle^{(1)} |j\rangle^{(2)}$. Suppose that Hamiltonian is of the form $\hat{H} = \hat{H}^{(1)}\hat{1}^{(2)} + 1^{(1)}\hat{H}^{(2)}$. For simplicity such a Hamiltonian is often written $\hat{H} = \hat{H}^{(1)} + \hat{H}^{(2)}$ with the identity operators implicit. Hamiltonians of this form have no interactions between the two systems. Show that if the individual system basis states $|i\rangle^{(1)}$ and $|j\rangle^{(2)}$ are eigenstates of the individual Hamiltonians $\hat{H}^{(1)}|i\rangle^{(1)} = E_{i_1}|i\rangle^{(1)}, \hat{H}^{(2)}|j\rangle^{(2)} = E_{j_2}|j\rangle^{(2)}$, then the eigenstates of the total Hamiltonian are the states $|i,j\rangle$ and their energy eigenvalues are $E_{i_1} + E_{j_2}$ that is $\hat{H}|i,j\rangle = \left(E_{i_1} + E_{j_2}\right)|i,j\rangle$.

3. In class we used separation of variables with the coordinate space representation of the Schrodinger equation to show that the two dimension harmonic oscillator had energy eigenvalues of $E = (N+1)\hbar\omega$ with degeneracy N+1 and $N = n_x + n_y = 0,1,2,3,\ldots..$.
    a. One can derive the same result more simply using the result of problem 2. In this problem, I want you to do this. Begin by showing that the Hamiltonian two dimensional harmonic oscillator can be written in the form of $\hat{H} = \hat{H}^{(x)} + \hat{H}^{(y)}$ (in the shorthand notation)

    b. Use a generalization of the same technique to for the three dimensional harmonic oscillator to show that the energy eigenvalues are $E = \left(N + \tfrac{3}{2}\right)\hbar\omega$ with degeneracy $\dfrac{(N+1)(N+2)}{2}$.

4. Consider the situation described in problem 2. Suppose that the system is in the state $|\psi\rangle = \tfrac{1}{2}\left(|1,1\rangle + \sqrt{2}\,|1,3\rangle + |2,1\rangle\right)$. Suppose we can individually measure the two systems. What is the probability that:
    a. System 1 is in state 1.
    b. System 1 is in state 2
    c. System 2 is in state 1
    d. System 2 is in state 3

e. System 1 is in state 1 & system 2 is in state 1.
f. System 1 is in state 2 & system 2 is in state 3.

Explain your reasoning.