# PHYS 402 Homework---Due Friday April 8

This homework assignment concerns a spin ½ system. The Hamiltonian for this system is of the form $\hat{H} = \frac{\Omega}{2}\hat{\sigma}_z + f(t)\hat{\sigma}_x$. Where f(t) is some time dependent function. This system can be realized in the lab by putting the spin in a constant magnet field in the z direction and a time dependent one in the x direction. We will work in the basis in of eigenstates of $\hat{\sigma}_z$. At t=0 the system in the spin up state, *i.e* $\left|\uparrow\right\rangle$.

1. Consider the case where $f(t) = \theta(t)\theta(T-t)\alpha$. That is the perturbation is of constant strength $\alpha$ for 0<t<T and zero elsewhere.
    a. Use first order perturbation theory to compute the state function for 0<t<T.
    b. Compute the probability of finding the particle in the down state ($\left|\downarrow\right\rangle$) as a function of time.
    c. From the form of this answer find an expression for the regime in which one expects perturbation theory to be valid. Express this in terms of $T, \alpha$, and Ω.

2. The preceding problem can be solved exactly: it is a precession problem similar to those we have considered before.
    a. Find the exact expression for the state as a function of time.
    b. Expand the exact solution as a Taylor series in α and show it yields the perturbative result.

3. Consider the case where $f(t) = \theta(t)\theta(T-t)\alpha$ t/T. That is the perturbation is of strength $\alpha t/T$ for 0<t<T and zero elsewhere.
    a. Use first order perturbation theory to compute the state function for 0<t<T.
    b. Compute the probability of finding the particle in the down state ($\left|\downarrow\right\rangle$) as a function of time.
    c. From the form of this answer find an expression for the regime in which one expects perturbation theory to be valid. Express this in terms of $T, \alpha$, and Ω.

4. Consider the case where $f(t) = \theta(t)\theta(T-t)\alpha$ sin(ωt). That is the perturbation is of strength sin(ωt). for 0<t<T and zero elsewhere.
    a. Use first order perturbation theory to compute the state function for 0<t<T.
    b. Compute the probability of finding the particle in the down state ($\left|\downarrow\right\rangle$) as a function of time.
    c. From the form of this answer find an expression for the regime in which one expects perturbation theory to be valid. Express this in terms of $T, \alpha, \omega$ and Ω.

5. Consider the case in problem 1) .
    a. Compute the state of the system to second order in perturbation theory.
    b. Compute the probability of finding the particle in the down state ($\left|\downarrow\right\rangle$) as a function of time.
    c. Verify that the exact solution expanded to second order in *a* gives this result.