

## PHYS 402 Homework---Due Monday April 1

1. Consider a particle of mass  $m$  confined to one dimensional infinite square well located between  $x=0$  and  $x=L$ .

Suppose that added to this a small perturbation of the form  $\hat{H}' = a\delta(x - L/4)$ .

- a. Calculate the first order shift in the energy.
- b. Estimate the second order shift in the energy. (You will need to do this numerically. Include enough terms in the sum to get approximate convergence.
- c. What is the condition to be considered small. Express this condition in term of  $a$ ,  $m$ ,  $L$  and  $\hbar$ .

2. Consider two distinguishable spin  $\frac{1}{2}$  particles whose spatial states are fixed and do not play a role in the problem.

The particles interact via the following Hamiltonian.  $\hat{H} = \frac{A}{\hbar^2} \hat{s}_1 \cdot \hat{s}_2 + \frac{B}{\hbar} \hat{s}_{1z}$ . If we assume that  $A \gg B$  then the

second term can be considered as a perturbation. The purpose of this problem is to compute the energies to second order in the perturbation.

- a. Write the unperturbed energies and state vectors. (Represent them in the  $|S m\rangle$  basis)
- b. Calculate the first order perturbation in the energies. Note that there is triply degenerate level and you must use degenerate perturbation theory. Fortunately you should find that the perturbation is already diagonal in the  $|S m\rangle$
- c. Compute the second order shift in energies.

3. Consider a particle of mass  $m$  confined to a two dimensional box between 0 and  $L$  in both the  $x$  and  $y$  directions.

That is  $V = \begin{cases} 0 & \text{for } 0 < x < L \text{ and } 0 < y < L \\ \infty & \text{otherwise} \end{cases}$ .

- a. Find the eigen energies. Note that the first excited state is doubly degenerate.
- b. Suppose that a perturbation of the form  $\hat{H}' = a \cos\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi y}{L}\right)$  is added to the Hamiltonian. Such a perturbation will split the degeneracy of the first excited states. Calculate the energies of these split states to first order in  $a$ .