PHYS 402 Homework---Due March 4

1. A Stern-Gerlach apparatus is set up to measure the component of spin of an electron along the direction in the x-z plane which makes an angle of θ relative to the z-axis. That is it is in the direction $\hat{n} = \cos(\theta)\hat{z} + \sin(\theta)\hat{x}$ A measurement is made and the electron us found to be spin up in this direction
   a. Compute the eigenvector of this state as given in the standard basis (oriented along z)
   b. Suppose following this measurement a second Stern-Gerlach apparatus oriented in the same direction measure the component of spin in the $\hat{n}$ direction. What is the probability it will be spin up?
   c. Suppose instead that following the initial measurement a second Stern-Gerlach apparatus oriented along the $\hat{x}$ direction measures the spin component. What is the probability it will be spin up?
   d. Suppose instead that following the initial measurement a second Stern-Gerlach apparatus oriented along the $\hat{y}$ direction measures the spin component. What is the probability it will be spin up?
   e. Suppose instead that following the initial measurement a second Stern-Gerlach apparatus oriented along the $\hat{z}$ direction measures the spin component. What is the probability it will be spin up?

2. Suppose two spin ½ particles combine to make a system and the state of the system (as expressed in a basis of total spin and total third component of spin is known to be $|\psi\rangle = \frac{1}{\sqrt{2}}|0,0\rangle + \frac{1}{2}|1,0\rangle + \frac{1}{2}|1,1\rangle$
   a. What is the probability that second particle is up?
   b. What is the probability that the first particle is down and the second particle is up?
   c. What is the probability that both particles are up?
   d. What is the probability that both particles are down?

3. Show that the most general rotationally invariant (ie. scalar) Hamiltonian for two spin ½ particles can be represented as $\hat{H} = a\hat{1} + b\hat{\vec{s}}_1 \cdot \hat{\vec{s}}_2$ where a and b are constants and $\hat{1}$ is the unit matrix. Hint: how many states are there? What degeneracy do you expect?