PHYS 402 Homework---Due February 25

1. Construct the matrices for $\hat{J}_x, \hat{J}_y$ and $\hat{J}_z$ for the states with j=3/2. Verify by explicit matrix multiplication that these 4 dimensional matrices satisfy the standard angular momentum commutation relations.

2. In class it was claimed that $\hat{R}(\vartheta) = e^{i\vartheta\hat{J}_z/\hbar}$ is the generic rotation operator about the z axis. If this is correct, it should be true that $\hat{R}(\vartheta)^{+}\,\hat{x}\,\hat{R}(\vartheta) = \cos(\vartheta)\,\hat{x} + \sin(\vartheta)\,\hat{y}$. Verify that this is true. The easiest way to do this is by differentiating both sides with respect to θ, verifying the relation there and then integrating. A useful first step in showing this is proving that $\frac{d\,e^{i\vartheta\hat{J}_z/\hbar}}{d\vartheta} = \frac{i\hat{J}_z}{\hbar}e^{i\vartheta\hat{J}_z/\hbar}$. Note also that here $\hat{J}_z = \hat{L}_z = \hat{x}\,\hat{p}_y - \hat{y}\,\hat{p}_x$

3. a) Show that $\left(\vec{n}\cdot\hat{\vec{\sigma}}\right)^2 = \hat{1}$ where $\vec{n}$ is a unit vector ($\vec{n}\cdot\vec{n} = 1$) and $\hat{1}$ is the identy matrix. The dot product has its normal meaning $\vec{n}\cdot\hat{\vec{\sigma}} \equiv n_x\hat{\sigma}_x + n_y\hat{\sigma}_y + n_z\hat{\sigma}_z$. b) Use the result in problem 3a) to show that $e^{i\vec{n}\cdot\hat{\vec{\sigma}}\vartheta} = \cos(\vartheta) + i\,\vec{n}\cdot\hat{\vec{\sigma}}\sin(\vartheta)$

4. Consider a spin ½ particle. The state of the system at anytime can always be represented as a normalized 2 dimensional vector $\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$. The magnetic moment of the particle is given by $\vec{\mu} = \mu_o\vec{\sigma}$ where μ_o is a numerical constant with the dimenension of magnetic moment. Assume that the system is placed in a magnetic field given by $\vec{B} = B_0\hat{z}$ (where $\hat{z}$ is a unit vector in the z direction). Suppose that at t=0 the state of the system is given by $\begin{pmatrix} a(0) \\ b(0) \end{pmatrix} = \begin{pmatrix} \cos(\vartheta/2) \\ \sin(\vartheta/2) \end{pmatrix}$ where θ is a parameter. a) Verify that the initial states is normalized. b) Show that $\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} e^{i\Omega t}\cos(\vartheta/2) \\ e^{-i\Omega t}\sin(\vartheta/2) \end{pmatrix}$ where $\Omega = \frac{B_0\mu_0}{\hbar}$. c) Show that $\langle\sigma_x\rangle = \sin(\vartheta)\cos(\Omega t)$, $\langle\sigma_y\rangle = -\sin(\vartheta)\sin(\Omega t)$ and $\langle\sigma_z\rangle = \cos(\vartheta)$.