PHYS 402 Homework---Due February 11

1. Consider the infinite spherical square well of radius *a* studied in class. We found the energies of the *l=0* states but showed that the *l*=1 states could only be solved numerically by finding the roots of a transcendental equation. Find a numerical estimate for the energies of the lowest three by solving this equation.

2. The spherical harmonics are a complete orthonormal set describing smooth angular functions. Completeness means than any smooth angular function may be written in the form $f(\vartheta,\phi)=\sum_{l,m} c_{l,m}\, Y_l^m(\vartheta,\phi)$. Use orthonormality to find the coefficients $c_{l,m}$ for the function $f(\vartheta,\phi)=\left(\sin(\vartheta)+\sin(2\vartheta)\right)\cos(\phi)$.

3. In class it was claimed that the spherical harmonics were orthonormal in the sense that $\int_0^{2\pi} d\phi \int_0^{\pi} d\vartheta \sin(\vartheta)\left(Y_{l'}^{m'}(\vartheta,\phi)\right)^* Y_l^m(\vartheta,\phi)=\delta_{l.l'}\delta_{m.m'}$ .Explicitly show that this holds for the subset $Y_2^2(\vartheta,\phi)$, $Y_2^1(\vartheta,\phi)$ and $Y_2^0(\vartheta,\phi)$ and $Y_1^0(\vartheta,\phi)$ (where the explicit forms of these are as given in the book or class)

4. Find the following matrix elements for states labeled $|l,m\rangle$;
   a. $\langle 1,2|L_z|2,2\rangle$
   b. $\langle 2,2|L_x|2,2\rangle$
   c. $\langle 2,2|L_x|2,1\rangle$
   d. $\langle 2,2|L_x|1,1\rangle$
   e. $\langle 2,2|L_x^2+L_y^2|2,2\rangle$