PHYS 402 Homework---Due Friday April 15

1. Consider a Hamiltonian which explicitly depends on time. At *t=0* the Hamiltonian is $\hat{H}_0$ and at *t=T* it is $\hat{H}_f$. Suppose that at *t=0* the system is in the ground state of $\hat{H}_0$. We have argued in that if the time variation of the Hamiltonian is very slow (adiabatic), then at *t=T* it will be in the ground state of $\hat{H}_f$. In general, this does not mean, however, that there is no probability of finding the state in the ground state of $\hat{H}_0$. Similarly we have argued that if the time variation is very fast, then at *t=T* the system will remain in the ground state of $\hat{H}_0$. Again this does not mean that that there is no probability of finding the state in the ground state of $\hat{H}_f$. Show that the probability that the system is in the ground state of $\hat{H}_0$ at *t=T* for adiabatic time variations is exactly the same as the probability that it is in the ground state of $\hat{H}_f$ for sudden ones.
2. A particle of mass *m* is in the ground state of a harmonic oscillator with natural frequency ω_0 at *t=0.* At *t=0* a perturbation of the form $\hat{H}' = \frac{1}{2} m\omega_0^2 x^2 (1 - e^{-t/\tau})$ is added on. Thus as $t \to \infty$ the system finds itself in a harmonic oscillator with a frequency of $\omega = \sqrt{2}\,\omega_0$.
    a. Find the state of the system at long time for the regime $\omega_0 \tau >> 1$ (you may neglect phases).
    b. Find the state of the system at long time for the regime $\omega_0 \tau << 1$ (you may neglect phases).
    c. For the regime $\omega_0 \tau >> 1$ what is the probability that the system at long times is in the ground state of the original harmonic oscillator.
    d. For the regime $\omega_0 \tau << 1$ what is the probability that the system at long times is in the ground state of the final harmonic oscillator.

3. A particle of mass *m* is in the ground state of an infinite spherical well of radius *R.* The walls of the well are slowly expanded to 2R. How much work does the particle to on the wall during this expansion?