PHYS 402 Homework---Due May 6

1. Consider scattering of a potential of the form $V(\vec{r}) = V_0\, e^{-\lambda\, r^2}$.
    a. Use the Born approximation to show that at sufficiently low energies and sufficiently small V_0, the scattering is isostropic . State the condition which specifies sufficiently low energies and sufficiently small V_0 for the Born approximation to be valid and for the potential to be isotropic.
    b. Compute the differential cross-section $\frac{d\sigma}{d\Omega}$ in the Born approximation
    c. Show that only phase shift contributing is the l=0 and find the phase shifts by direct comparison with the scattering amplitude. (You do not need to solve the Schrodinger equation to do this.
    d. In computing the result in part b. you computed a scattering amplitude in the Born approximation which, if done correctly, was entirely real. The optical theorem says that the total cross-section is given by $\sigma = \frac{4\pi\, \mathrm{Im}\left(f(\theta=0)\right)}{k}$. Clearly the total cross section in a. is nonzero. How do you reconcile these two facts.

2. Show that in the regime where the Born Approximation is valid, the differential cross-section for back-scattering ($\theta = \pi$) at an incident energy E_0 is the same as differential cross-section for scattering in the perpendicular direction ($\theta = \pi/2$) at an incident energy $2E_0$. (Hint: think about momentum transfer).

3. In class we showed that in the Born approximation scattering from a Yukowa potential of the form $V(\vec{r}) = C\frac{e^{-\lambda\, r/a}}{r}$, yields a scattering amplitude of $f = -\left(\frac{2m}{\hbar^2}\right)\frac{C}{q^2 + \left(\frac{1}{a}\right)^2}$ where q is the magnitude of the momentum transfer. We have also argued that at high energies the scattering should be forward peaked. The purpose of this problem is to demonstrate this explicitly.
    a. Show that the maximum of the differential cross-section is maximal in the forward direction ($\theta = 0$) and is is given by $\left.\frac{d\sigma}{d\Omega}\right|_{\max} = \frac{4m^2C^2a^4}{\hbar^4}$.
    b. Show that the angle at which the differential cross-section is reduced from the maximum by a factor of four is given by $\theta = 2\sin^{-1}\left(\frac{1}{2k\,a}\right) \approx \frac{1}{k\,a}$ where the last equality holds for $k\,a >> 1$. Explain why this result implies extreme forward peaking at very high energies.