Phys 402
Spring 2009
Homework 3
Due Friday, February 20, 2009 @ 9 AM

1. Griffiths, 2nd Edition, Problem 6.1 — **First-order perturbation theory with a delta-function in an infinite square well. Eigen-energy and ground state eigenfunction.**

2. Griffiths, 2nd Edition, Problem 6.4 (a) only — **Second-order perturbation theory for the above. Trick: Try writing $\dfrac{1}{n^2 - m^2}$ as $\dfrac{A}{n-m} + \dfrac{B}{n+m}$ to help sum the infinite series.**

3. Griffiths, 2nd Edition, Problem 6.5 (a) only — **Second-order perturbation theory for the charged harmonic oscillator in an electric field.**

4. Griffiths, 2nd Edition, Problem 6.11 — **Fine Structure constant**

Extra Credit #4 — **Degenerate Perturbation Theory**
       Stark Effect in Hydrogen. In 1913 Stark observed a splitting of the Hydrogen Balmer series lines by applying an electric field E = 100,000 V/cm.
  a) **Write down the perturbing Hamiltonian H' for the electron. Neglect spin in this problem.**
  b) **Calculate the change in energy of the ground state of Hydrogen to first order.**
  c) **Consider the n=2 states of Hydrogen. Find the new energies to first order.**
  d) **Calculate the new n=2 eigenfunctions.**
  e) **Roughly calculate the resulting splitting of the Balmer series H$_{\alpha}$ line in an electric field of 100,000 V/cm.**

1. Consider the one-dimensional <u>anharmonic</u> oscillator with Hamiltonian
$$H = H_0 + K' x^4$$
and the unperturbed Hamiltonian $H_0 = (p_x^2 / 2m) + k\, x^2 / 2$.

  a) Write down an expression for the first order correction to the eigenenergies.
  b) Evaluate the expression for the first order correction to the eigenenergy of the nth harmonic oscillator state. Hint: Look back at Eqs. [2.69] and [2.66].

28. Eigenfunctions for a rigid dumbbell rotating about its center have a ϕ dependence of the form $\psi(\phi) = Ae^{im\phi}$, where m is a quantum number and A is a constant. Which of the following values of A will properly normalize the eigenfunction?

(A) $\sqrt{2\pi}$

(B) 2π

(C) $(2\pi)^2$

(D) $\dfrac{1}{\sqrt{2\pi}}$

(E) $\dfrac{1}{2\pi}$

Physics GRE Quantum Mechanics!

28. A system is known to be in the normalized state described by the wave function

$$\psi(\theta, \varphi) = \frac{1}{\sqrt{30}}(5 Y_4^3 + Y_6^3 - 2 Y_6^0),$$

where the $Y_\ell^m(\theta, \varphi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is

(A) 0

(B) $\dfrac{1}{15}$

(C) $\dfrac{1}{6}$

(D) $\dfrac{1}{3}$

(E) $\dfrac{13}{15}$

49. The Hamiltonian operator in the Schrödinger equation can be formed from the classical Hamiltonian by substituting

(A) wavelength and frequency for momentum and energy
(B) a differential operator for momentum
(C) transition probability for potential energy
(D) sums over discrete eigenvalues for integrals over continuous variables
(E) Gaussian distributions of observables for exact values

50. The state of a quantum mechanical system is described by a wave function ψ. Consider two physical observables that have discrete eigenvalues: observable A with eigenvalues $\{\alpha\}$, and observable B with eigenvalues $\{\beta\}$. Under what circumstances can all wave functions be expanded in a set of basis states, each of which is a simultaneous eigenfunction of both A and B?

(A) Only if the values $\{\alpha\}$ and $\{\beta\}$ are nondegenerate
(B) Only if A and B commute
(C) Only if A commutes with the Hamiltonian of the system
(D) Only if B commutes with the Hamiltonian of the system
(E) Under all circumstances

77. Consider a heavy nucleus with spin $\frac{1}{2}$. The magnitude of the ratio of the intrinsic magnetic moment of this nucleus to that of an electron is

(A) zero, because the nucleus has no intrinsic magnetic moment
(B) greater than 1, because the nucleus contains many protons
(C) greater than 1, because the nucleus is so much larger in diameter than the electron
(D) less than 1, because of the strong interactions among the nucleons in a nucleus
(E) less than 1, because the nucleus has a mass much larger than that of the electron

96. A particle of mass M is in an infinitely deep square well potential V where

$$V = 0 \quad \text{for } -a \le x \le a, \text{ and}$$
$$V = \infty \quad \text{for } x < -a, a < x.$$

A very small perturbing potential V' is superimposed on V such that

$$V' = \epsilon\left(\frac{a}{2} - |x|\right) \qquad \text{for } \frac{-a}{2} \le x \le \frac{a}{2}, \text{ and}$$
$$V' = 0 \qquad \text{for } x < \frac{-a}{2}, \frac{a}{2} < x.$$

If $\psi_0, \psi_1, \psi_2, \psi_3, \ldots$ are the energy eigenfunctions for a particle in the infinitely deep square well potential, with ψ_0 being the ground state, which of the following statements is correct about the eigenfunction ψ_0' of a particle in the perturbed potential $V + V'$?

(A) $\psi_0' = a_{00}\psi_0,\ a_{00} \ne 0$

(B) $\psi_0' = \displaystyle\sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all odd values of n

(C) $\psi_0' = \displaystyle\sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all even values of n

(D) $\psi_0' = \displaystyle\sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} \ne 0$ for all values of n

(E) None of the above