Lecture 1 Summary
Phys 402

We started with a brief review of classical physics. Kinematics is the description of motion. Classically, a particle is a point-like object that follows a well-defined trajectory given by $\vec{x}(t)$. At any instant it has a velocity vector $\vec{v}(t)$ and acceleration vector $\vec{a}(t)$. Dynamics is the study of why motion occurs. It is summarized by Newton's second law of motion, relating a kinematic quantity to the net unbalanced force acting on the particle of mass m: $\vec{a} = \vec{F}_{net} / m$. Conservative forces can be derived from a scalar potential as $\vec{F} = -\vec{\nabla}V$, and manipulation of the scalar potential is often easier than solving problems directly with the vector force. (In quantum mechanics all microscopic forces are conservative.)

In quantum mechanics, kinematics plays essentially no role, since the concept of trajectory is not compatible with the uncertainty principle for position and momentum. Dynamics survives in the form of the time-dependent Schrödinger equation. We also must adopt a probabilistic approach to understanding quantum mechanical systems, as expressed through the wavefunction $\Psi(\vec{x},t)$. We interpret the absolute square of the complex wavefunction $|\Psi(\vec{x},t)|^2$ as a probability density. The quantity $|\Psi(\vec{x},t)|^2 d^3x$ is the probability of finding the particle within a small volume d^3x of the point $\vec{x}$ in 3D. Probability is a dimensionless number between 0 and 1, inclusive. Probability and statistics play very important roles in QM, and the basic concepts related to probability distribution functions (PDFs) are worth reviewing on your own.

The time-dependent Schrödinger equation in one dimension is:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi$$. Here $\hbar = 1.05\ldots \times 10^{-34}$ J-s is Planck's constant divided by 2π. Planck's constant was originally introduced as a fitting parameter to describe the spectrum of radiation coming out of a black body radiator. Since then it has also been recognized as the quantum of angular momentum and of action.

All measurements are represented by corresponding operators in QM. The operator corresponding to the x-component of momentum is $p_x = -i\hbar \partial / \partial x$. One way to describe the outcome of experiments in QM is through the expectation value. The expectation value of an operator $\hat{D}(x, p_x)$ is $\langle D \rangle = \int \Psi^*(\vec{x})\hat{D}(x, p_x)\Psi(\vec{x})d^3x$. The interpretation of an expectation value is that it represents the average of measurements of $\hat{D}(x, p_x)$ taken on a large number of identically prepared systems, each of which is prepared in state Ψ. It is not the average of many repeated measurements on one system that had been initially prepared in the state Ψ. Note that the expectation value is just a single scalar measure of what is typically a full and continuous probability distribution function of experimental outcomes. As such it can sometimes be misleading.

Compatible observables are those that can be determined simultaneously without corrupting information gained about each other. In other words, the outcome of measurements of two compatible operators is independent of the order in which they are measured. It turns out that angular momentum squared L^2 and the z-component of

angular momentum L_z are compatible observables (this is true for L^2 and any other single component of angular momentum). Hence if one has a Hydrogen atom (for example) prepared in state Ψ, then $L^2 L_z \Psi = L_z L^2 \Psi$. In other words, determining the eigenvalue of L_z has no influence on the subsequent determination of L^2, and vice versa. This is best summarized as a property of the operators (independent of any and all wavefunctions Ψ) as $L^2 L_z - L_z L^2 = \left[L^2, L_z\right] = 0$, where the bracket is called the commutator of the two operators. One can always construct simultaneous eigenfunctions of compatible operators. For example, the full hydrogen atom wavefunction is a simultaneous eigenfucntion of 3 compatible operators, L^2, L_z, and the Hamiltonian operator.

Incompatible operators on the other hand destroy information obtained by previous measurements with other incompatible operators, hence they do not commute. One cannot form a simultaneous eigenfunction of two or more incompatible operators. This incompatibility leads to an uncertainty principle involving the outcomes of subsequent measurements with those incompatible operators (call them A and B):

$\sigma_A^2 \sigma_B^2 \geq \left(\dfrac{\langle[A,B]\rangle}{2i}\right)^2$ {Griffiths [3.62]}, where $\sigma_A^2 = \left\langle\left(A - \langle A\rangle\right)^2\right\rangle$ is essentially the

variance (a measure of the width) of the distribution of outcomes of measurement of A. For the case of incompatible operators x and p_x this reduces to the familiar

result $\sigma_x \sigma_{p_x} \geq \hbar/2$.

A separate document discusses the postulates of QM.