Physics 401 Homework 9---Due November 11

The first three problems are based on the time-independent Schrödinger equation for an attractive potential in terms of dimensionless variables: $\left(\frac{\partial^2}{\partial\xi^2} + \rho^2 f(\xi)\right)\psi(\xi) = -\varepsilon\psi(\xi)$ where

$V(x) = -V_0 f(\frac{x}{x_0})$, $\xi \equiv \frac{x}{x_0}$, $\rho^2 \equiv \frac{2mV_0 x_0^2}{\hbar^2}$, $\varepsilon \equiv \frac{2mE x_0^2}{\hbar^2}$.

1. As ρ becomes large the well becomes deep and supports many bound states. The lowest lying states are well-localized at the bottom of the well where the well looks harmonic.

   a. From this you should be able to show that in the limit of large ρ the ground state dimensionless energy, ε_0 is given approximately by $\varepsilon_0 \approx -\rho^2 f(\xi_{\max}) + \rho\sqrt{\frac{-f''(\xi_{\max})}{2}}$ where $\xi_{\max}$ is the value of ξ which maximizes f (*i.e.* miminimizes the potential which is attractive).

   b. Show that the excitation energy of the first excited state in dimensionless units is given approximately by $\varepsilon_1 - \varepsilon_0 \approx \rho\sqrt{-2f''(\xi_{\max})}$.

   The approximations in a. and b. should become more an accurate as ρ increases and become exact in the $\rho \to \infty$ limit.

   c. Consider the case where $f(\xi) = \frac{1}{\cosh(\xi)}$. Show that the approximate ground state energy and excitation energy of the first excited state in dimensionless units is given by $\varepsilon_0 \approx -\rho^2 + \sqrt{\frac{1}{2}}\rho \quad \varepsilon_1 - \varepsilon_0 \approx \sqrt{2}\rho$

2. Numerically solve for the ground state for the for the potential with $f(\xi) = \frac{1}{\cosh(\xi)}$. Determine the bound state energies by adjusting the energy until the wave function has a very small exponentially growing part. Plot the approximate wave function. You may find the mathematica notebook on the course website useful.
   a. Do this analysis for the case where $\rho = 10$.
   b. Do this analysis for the case where $\rho = 100$.

3. Verify that the energies obtained in 2a. and 2b. are close to those given by the approximations in 1c. Which case works better in terms of number of accurately predicted digits? Why?

Griffths: 2.30, 2.52