Physics 401 Homework 4---Due October 14

This homework assignments concerns the quantum mechanic harmonic oscillator. The wave functions ψ_n refers to the nth eigenfunction of this system.

1. Use ladder operators to show that:

   a. $\langle \psi_{n+1} | \hat{x} \psi_n \rangle = \sqrt{\frac{\hbar(n+1)}{2m\omega}}$

   b. $\langle \psi_{n-1} | \hat{x} \psi_n \rangle = \sqrt{\frac{\hbar n}{2m\omega}}$

   c. $\langle \psi_j | \hat{x} \psi_k \rangle = 0$ unless $k = j \pm 1$

   d. $\langle \psi_{n+1} | \hat{p} \psi_n \rangle = i\sqrt{\frac{\hbar m\omega(n+1)}{2}}$

   e. $\langle \psi_{n-1} | \hat{p} \psi_n \rangle = -i\sqrt{\frac{\hbar m\omega n}{2}}$

   f. $\langle \psi_j | \hat{p} \psi_k \rangle = 0$ unless $k = j \pm 1$

2. Suppose that at t=0 $\psi(x) = \sqrt{\frac{1}{3}}(\psi_1(x) + \psi_2(x) + \psi_3(x))$, show that
   $\psi(x,t) = \sqrt{\frac{1}{3}} e^{-i\omega t/2}\left(\psi_1(x)e^{-i\omega t} + \psi_2(x)e^{-i2\omega t} + \psi_3(x)e^{-i3\omega t}\right)$
3. Use the results of 1. and 2. to compute $\langle x \rangle$ and $\langle p \rangle$ as a function of time for the state in 2. Show that your result follows the classical equations of motion for a Harmonic oscillator.

4. Use Ehrenfest's theorem to show $\langle x \rangle$ and $\langle p \rangle$ always follows the classical harmonic oscillator equations of motion.

Griffths: 2.10