Due December 2

1. This problem concnerns the momentum representation.

   a. Show that

$$\hat{x}|p\rangle = -i\hbar \frac{d}{dp}|p\rangle$$

Hint:Look at the ovelap between two momentum operators, insert the identity operator in an appropriate place and use the following facts:

$$\hat{1} = \int dx |x\rangle\langle x| \qquad \langle x|p\rangle = \frac{e^{ipx}}{\sqrt{2\pi\hbar}}$$

   b. Show that the explict x operator im momentum on the momentum space wave functionϕ(p) is given by

$$\hat{x}\phi(p) = i\hbar\phi'(p)$$

   c. Show that momentum space Schrodinger equation is given by

$$\left(\frac{p^2}{2m} + V(i\hbar\partial_x)\right)\phi(p,t) = i\hbar\frac{\partial\phi(p,t)}{\partial t}$$

2. The purpose of this problem is to show that the momentum operator is the generator of translations. We will do this in two ways: acting on a wavefunction and on an abstract eigentste of x.

   a. Show that

$$e^{i\hat{p}a/\hbar}\psi(x) = \psi(x+a)$$

Hint: Differentiate both sides with respect to a. demonstrate the validity and then integrate.

   b. Show that

$$e^{i\hat{p}a/\hbar}|x\rangle = |x-a\rangle$$