QUANTUM PHYSICS I

PROBLEM SET 6

due November 29th

1. Center-of-mass versus relative motion

Consider a system made out two different particles with masses m and M interacting through a potential dependent only on the distance between them $V = V(|\mathbf{r}_1 - \mathbf{r}_2|)$. If we denote the position of each particle by the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ the hamiltonian of the system is:

$$\hat{H} = \frac{\hat{\mathbf{p}}_1^2}{2M} + \frac{\hat{\mathbf{p}}_2^2}{2m} + V(|\mathbf{r}_1 - \mathbf{r}_2|). \tag{1}$$

a) Show that by using the relative motion and center-of-mass variables:

$$\begin{aligned} \mathbf{R} &= \frac{M\mathbf{r}_1 + m\mathbf{r}_2}{M+m} \\ \mathbf{R} &= \mathbf{r}_1 - \mathbf{r}_2 \end{aligned} \tag{2}$$

the hamiltonian can be written as a sum of one operator dependent on $\mathbf{r}$ and another dependent on bfR only:

$$\hat{H} = \frac{\hat{\mathbf{p}}_r^2}{2\mu_1} + \frac{\hat{\mathbf{p}}_R^2}{2\mu_2} + V(r). \tag{3}$$

Find $\mu_{1,2}$ as a function of m and M.

b) Show that the ansatz $\psi(bfr, \mathbf{R}) = \psi_{CM}(\mathbf{R})\phi(\mathbf{r})$ separates the Schrodinger equation into an equation for a free particle for the center-of-mass motion and the equation for *one* particle in an extrenal potential $V(r)$ for the relative motion. Show that the energies for the center-of-mass and relative motion add to give the total energy of the system.

2. Harmonic oscillator inside a box

Find the energy levels of a particle in three dimensions subject to the potential

$$V(x,y,z) = \begin{cases} \frac{m\omega^2}{2}(x^2+y^2), & \text{for } 0 < z < L \\ \infty, & \text{otherwise} \end{cases} \tag{4}$$