Current-Voltage relations : "metal" – "Conductor" – "metal"

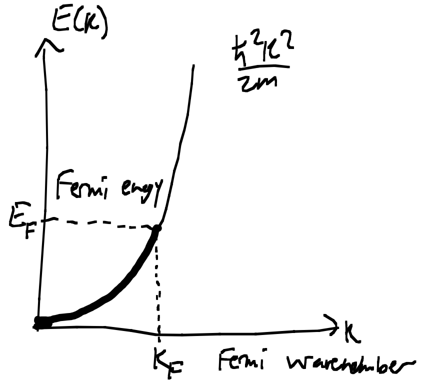
Our model for a metal is a noninteracting "gas" of free electrons

Using this approx, charge current

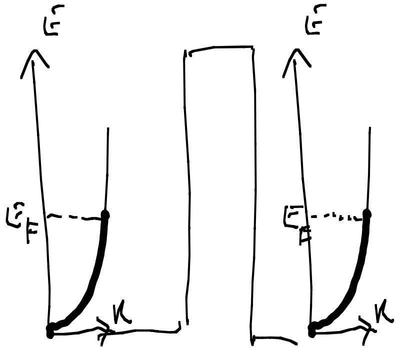
$$I_{1D} = \overset{\overset{\text{fundamental charge}}{\downarrow}}{q}\, \underset{\underset{\text{electron density}}{\nearrow}}{n}\, v_{avg} = q\,\frac{\sum_{i=1}^{N} V_i T_i}{\underset{\underset{\text{Size of metal lead}}{\nearrow}}{L}} = q \underset{\text{"contributing states"}}{\sum} \frac{\hbar k}{m} T(k) \Big/ L$$

what are "contributing states"?

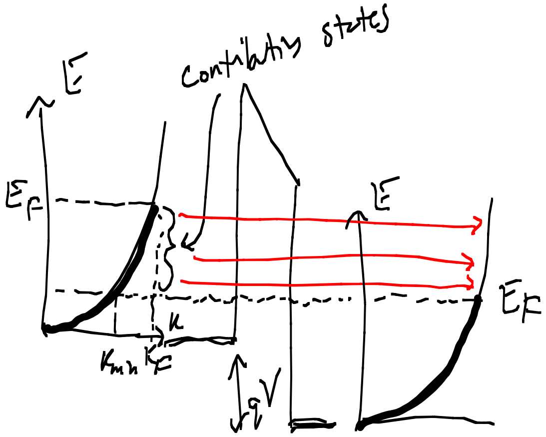
1. Available states $E = \frac{\hbar^2 k^2}{2m}$ "metal"
2. Occupied states $(T=0)$
    a. Minimization of energy
    b. Pauli exclusion principle



3. Asymmetry: $V=0,\ I=0$!



$V \neq 0$
$I \neq 0$



$$E + qV > E_F$$

$$\frac{\hbar^2 k^2}{2m} > E_F - qV$$

$$k > k_{min} = \sqrt{\frac{2m\,(E_F - qV)}{\hbar^2}}$$

Density of States

How much of k-space does each electron state take up?

Model infinite size of metal "leads" by using periodic boundary conditions:

$$\Psi(x) = \Psi(x+L)$$

$$e^{ikx} = e^{ik(x+L)} = e^{ikx}\, e^{ikL} \rightarrow \text{KL} = \text{integer multiples of } 2\pi$$

So successive states are separated by

$$\Delta K = \frac{2\pi}{L} \quad \text{(region of K-space occupied by 1 state)}$$

Our sum then becomes

$$I_{1D} = q \sum_{\substack{\text{Contrib.} \\ \text{States}}} \frac{\hbar K}{m} T(K) \frac{\Delta K}{2\pi} \longrightarrow \overset{\text{"spin degeneracy"}}{\overset{\downarrow}{2}} q \int_{K_{min}}^{K_F} \frac{\hbar k}{m} T(K,V) \frac{dK}{2\pi}$$

For small V

In this regime, $k_{min} \lesssim k_F$ so contributing states all have $E \sim E_F$. Therefore, we can approximate $T(E,V) \simeq T(E_F,V)$ (a constant). This allows us to take T out of the integral:

$$I_{1D} \cong 2q\frac{\hbar}{m}\frac{T}{2\pi}\int_{k_{min}}^{k_F} k\,dk = \frac{q\hbar T}{m\pi}\frac{k^2}{2}\Big|_{k_{min}}^{k_F}$$

$$= \frac{q\hbar T}{2\pi m}\left(k_F^2 - \frac{2m(E_F - qV)}{\hbar^2}\right)$$

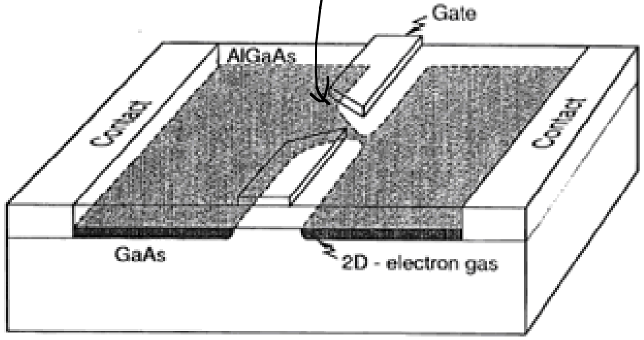
$$= \frac{q\hbar T}{2\pi m}\left(\frac{2mqV}{\hbar^2}\right) \qquad \left(\text{since } E_F = \frac{\hbar^2 k_F^2}{2m}\right)$$

$$= 2\underbrace{\frac{q^2}{h}}_{\text{"quantum of conductance"}}TV$$

$T \leq 1$ so conductance is limited by $\frac{2q^2}{h}$!

"Quantum Point Contact"

1-D constriction from electrostatic repulsion of gate



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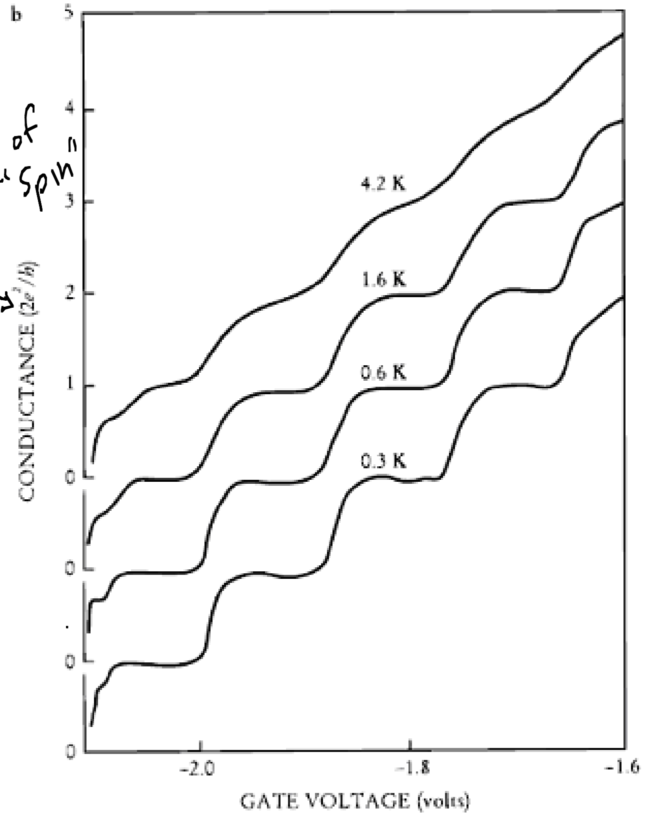
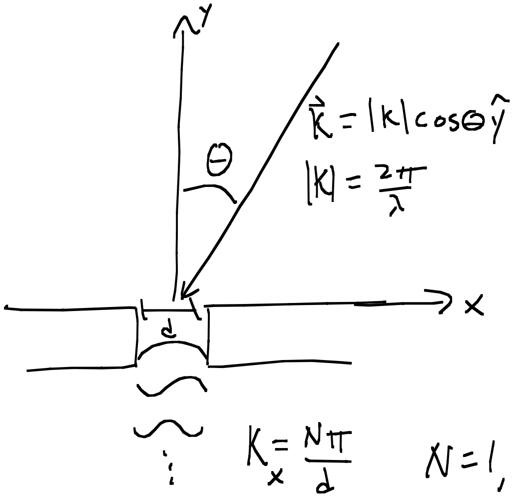


factor of 2 from "spin"

Steps in conductance as constriction widens and becomes equivalent to many parallel 1-D conductors

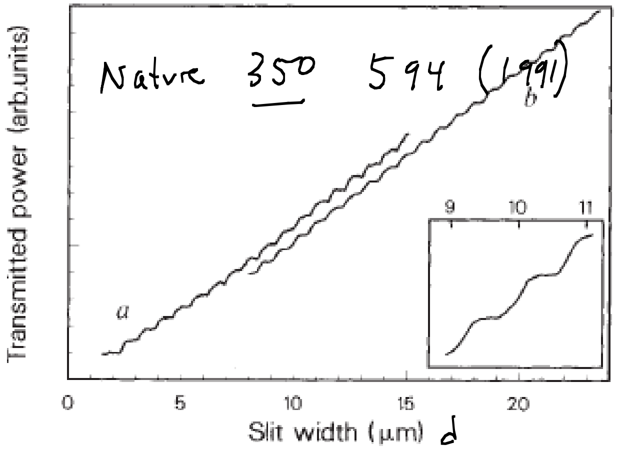
Optical "Quantized Conduction"



$\vec{k} = |k|\cos\theta\,\hat{y} + |k|\sin\theta\,\hat{x}$

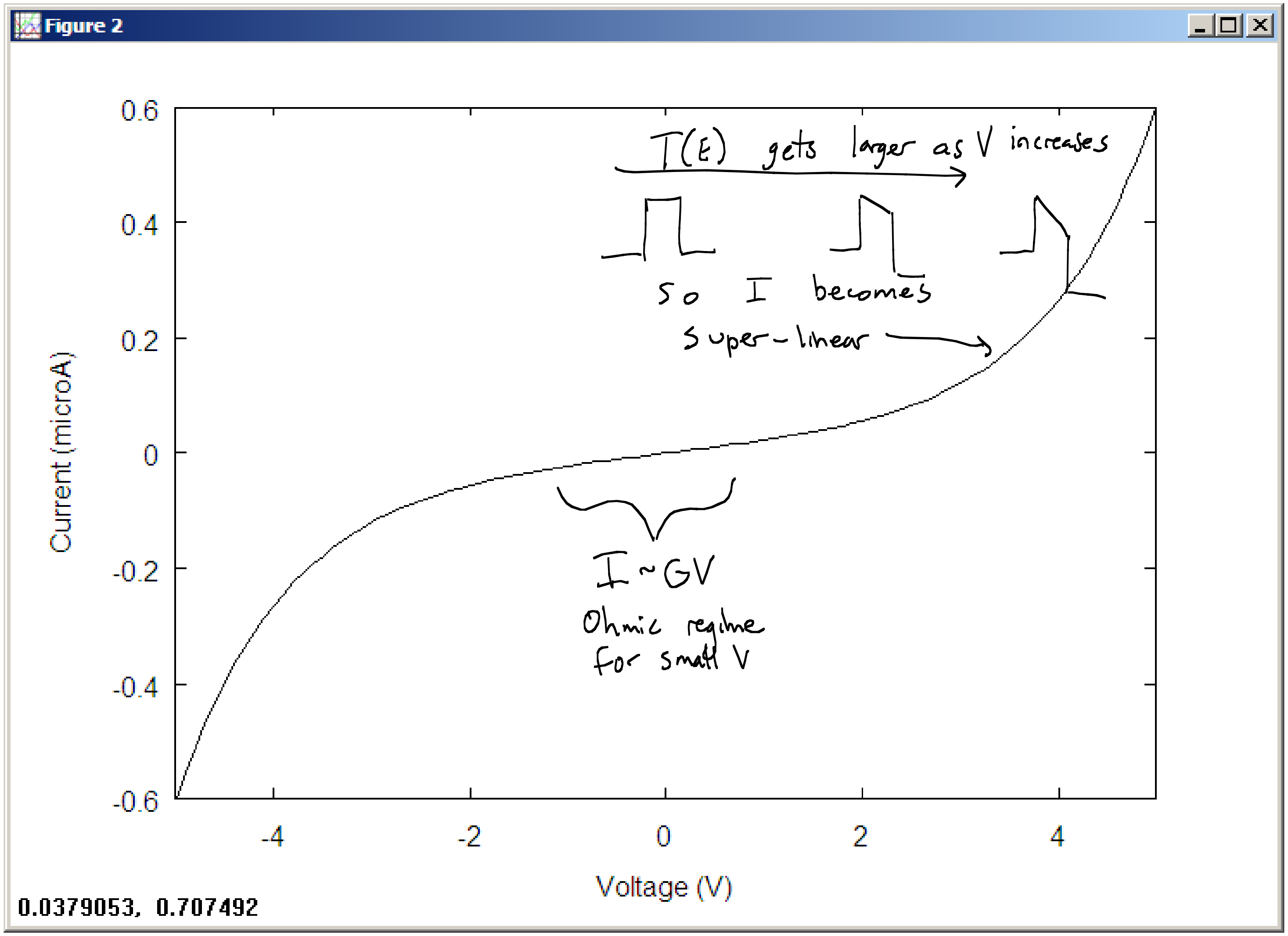
$|k| = \frac{2\pi}{\lambda}$

$K_x = \frac{N\pi}{d}$ $N = 1, 2, 3 \ldots$



Nature 350 594 (1991)

Numerical Results: Single barrier "Tunnel Junction"



0.0379053, 0.707492

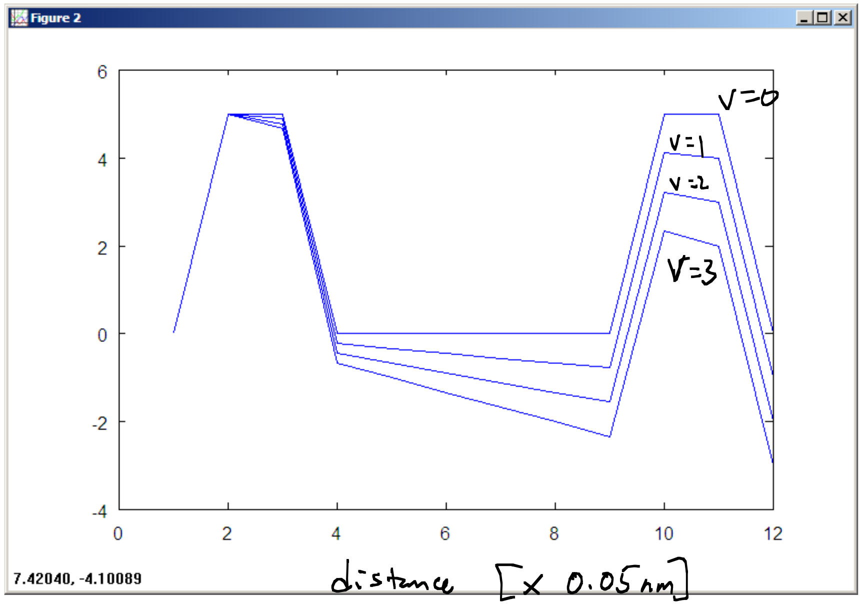
T(E) gets larger as V increases

So I becomes super-linear

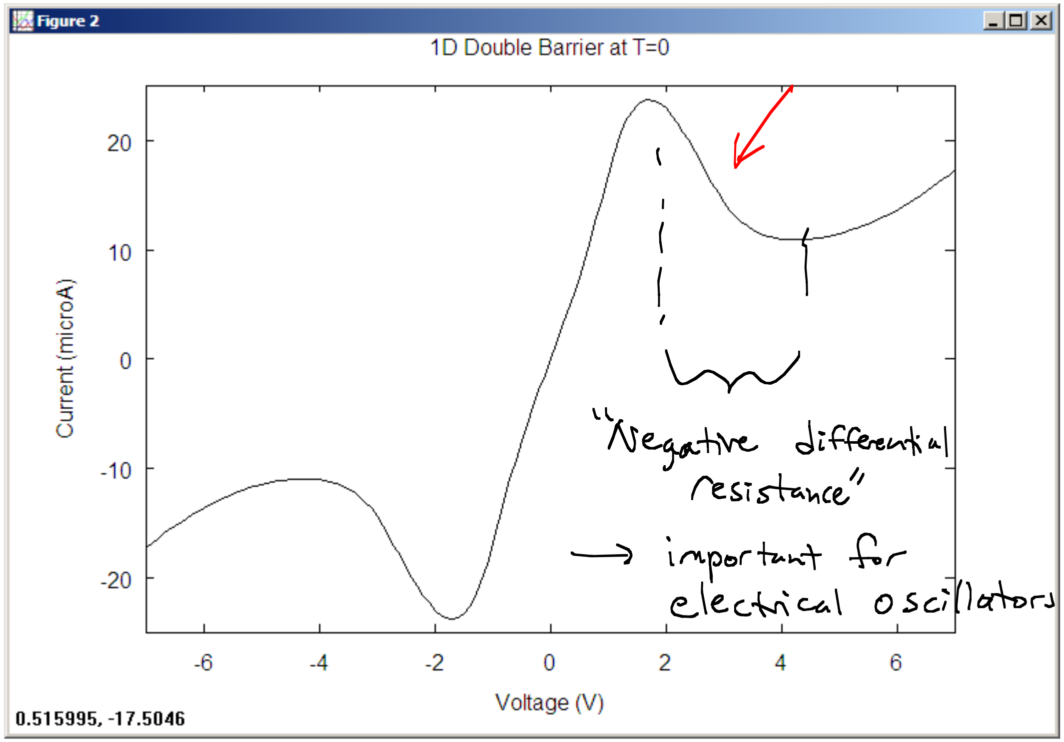
$I \sim GV$

Ohmic regime for small V

"Double barrier resonant tunneling diode"

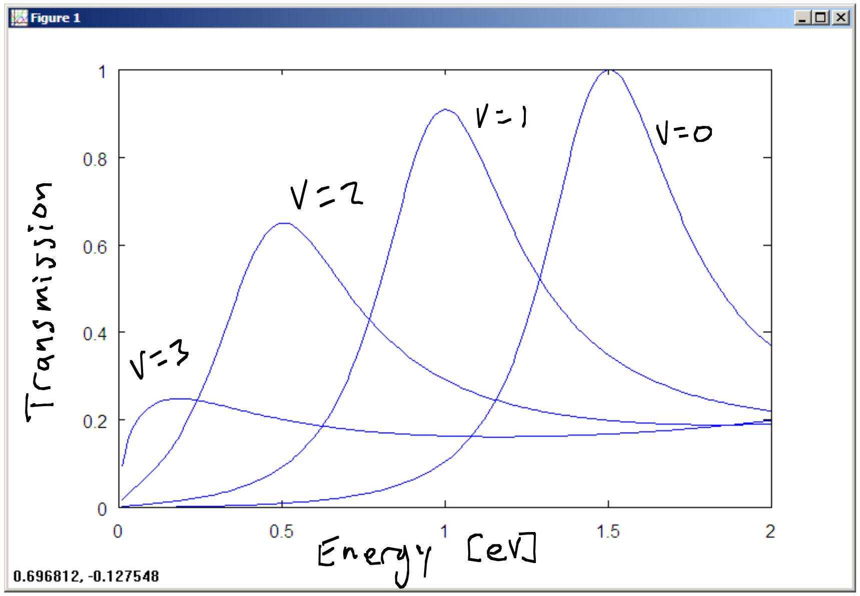


$(E_F = 2\,eV)$



"Negative differential resistance"

→ important for electrical oscillators



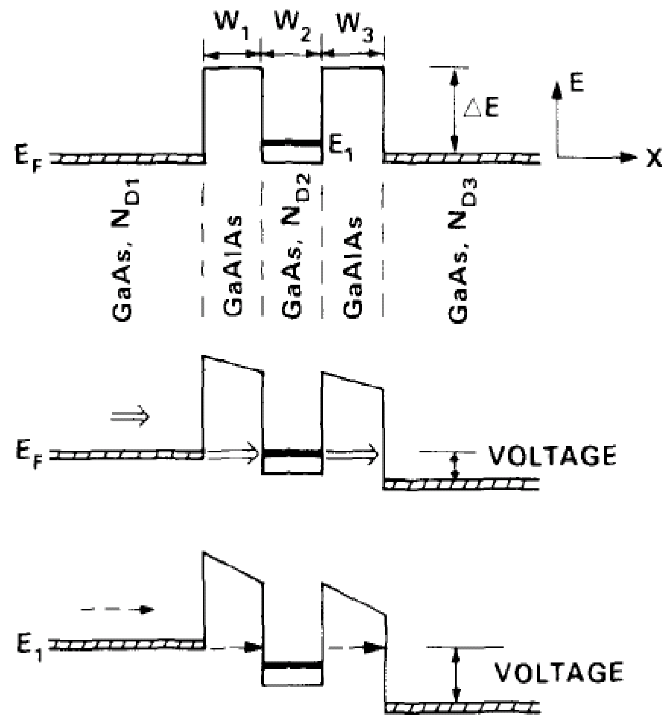
← resonant quasi-bound state pulled below E=0 as V gets larger – T(E) decreases.

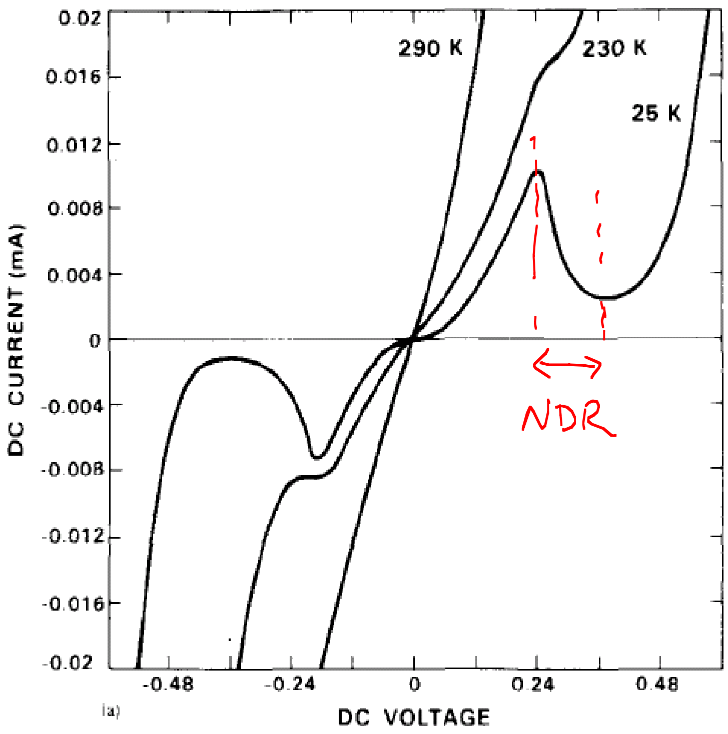
Experimental evidence for Negative differential Resistance

Resonant tunneling through quantum wells at frequencies up to 2.5 THz

T. C. L. G. Sollner, W. D. Goodhue, P. E. Tannenwald, C. D. Parker, and D. D. Peck
Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts 02173

588 Appl. Phys. Lett. **43** (6), 15 September 1983





(a)