## 1. Finite differences solution of the harmonic oscillator

```
clear; close all;

c=2.998e10;%cm/s
hbar=6.582e-16; %in eV*sec
m=5.11e5/c^2; %in eV/c^2

N=500; %size of matrix
xmax=6e-7;
x=linspace(-xmax,xmax,N);

dx=2*xmax/N;
tx=hbar^2/(2*m*dx^2);
omega=1e15;
V=(m/2*omega^2)*x.^2; %harmonic oscillator potential

H=tx*(diag(2*ones(N,1))+diag(-1*ones(N-1,1),1)+diag(-1*ones(N-1,1),-1))+diag(V);

[v,d]=eig(H);
E=diag(d);

figure(1);
plot(x*1e7,V,'k')
xlabel('distance [nm]'); ylabel('potential energy [eV]')
title(['Harmonic Oscillator, \omega=' sprintf('%.1e',omega) 'rad/s'] )
axis([-6 6 0 20])

%PLOT THE PARTICLE DENSITIES FROM LOWEST WAVEFUNCTIONS
figure(2)
plot(x*1e7, conj(v(:,1:3)).*v(:,1:3))

title(['Harmonic Oscillator, \omega=' sprintf('%.1e',omega) 'rad/s: electron density
distributions'] )
xlabel('distance [nm]'); ylabel('\Psi * \Psi')

figure(3);
MAXEIG=10; %highest quantum number to plot
ns=0:(MAXEIG-1);
plot(ns, E(ns+1),'*');
hold on;
plot(ns, hbar*omega*(ns+1/2), 'o')
legend('numerical','analytic')
axis([0 MAXEIG 0 MAXEIG*hbar*omega]);
title(['Harmonic Oscillator, \omega=' sprintf('%.1e',omega) 'rad/s: energy
eigenvalues'] )
xlabel('quantum number n'); ylabel('Energy eigenvalue [eV]')

ns=[9 49 99]; %quantum numbers
fignum=4;
for n=ns %this can be done without a ‘for’ loop by repeating the lines below
  figure(fignum);
  xmax_cl=sqrt(E(n+1)/((m*omega^2)/2)); %classical turning point
  plot(x*1e7,(omega/pi)*dx*real(xmax_cl*omega*sqrt(1-(x/xmax_cl).^2)).^(-1),'r');
  psistarpsi=conj(v(:,n+1)).*v(:,n+1); %quantum probability density
  hold on; plot(x*1e7, psistarpsi,'b'); hold off
  axis([-xmax*1e7 xmax*1e7 0 max(v(:,n+1))^2])
  title(['Comparison of quantum to classical probability density, n=' int2str(n)] );
  xlabel('distance [nm]'); ylabel('Probability')
  fignum=fignum+1;
end
```

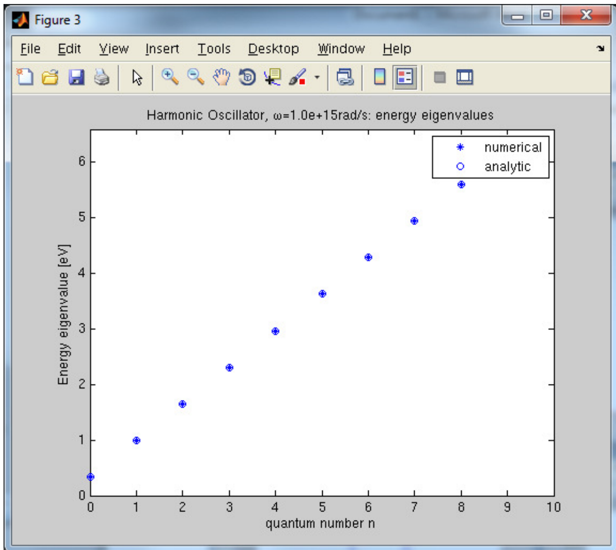
Figure 3
File Edit View Insert Tools Desktop Window Help
Harmonic Oscillator, ω=1.0e+15rad/s: energy eigenvalues
numerical
analytic
Energy eigenvalue [eV]
quantum number n

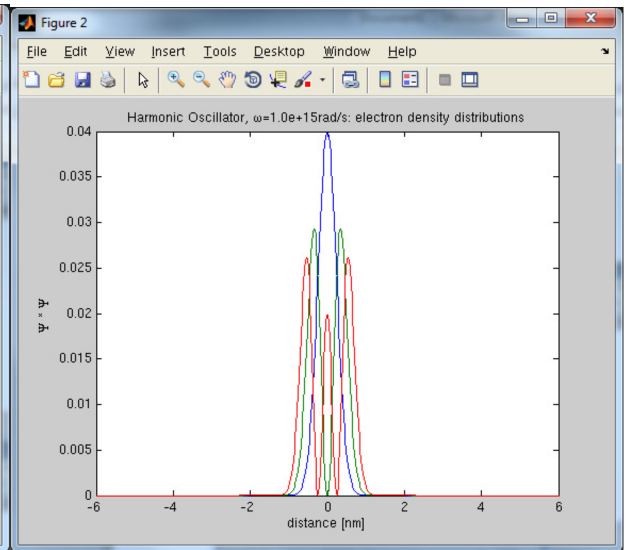
Figure 2
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Harmonic Oscillator, ω=1.0e+15rad/s: electron density distributions
Ψ * Ψ
distance [nm]

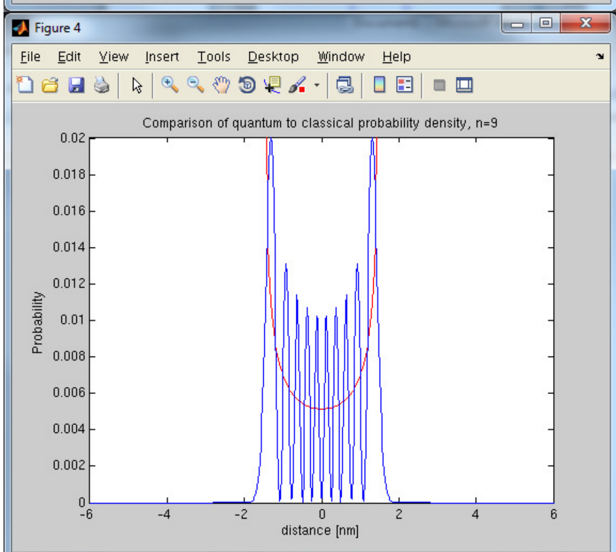
Figure 4
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Comparison of quantum to classical probability density, n=9
Probability
distance [nm]

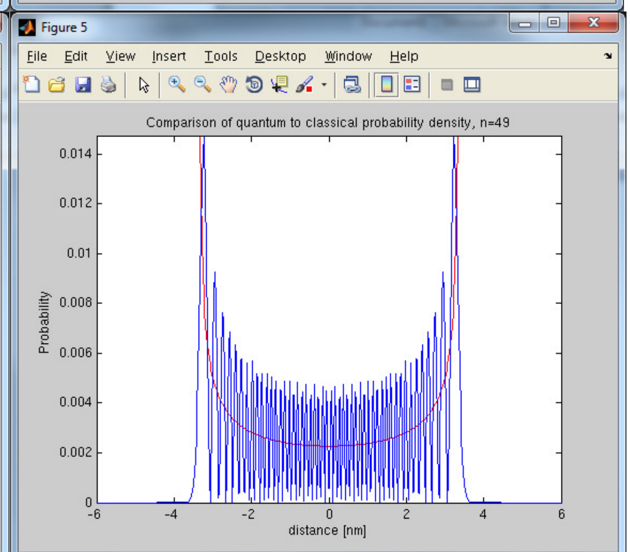
Figure 5
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Comparison of quantum to classical probability density, n=49
Probability
distance [nm]

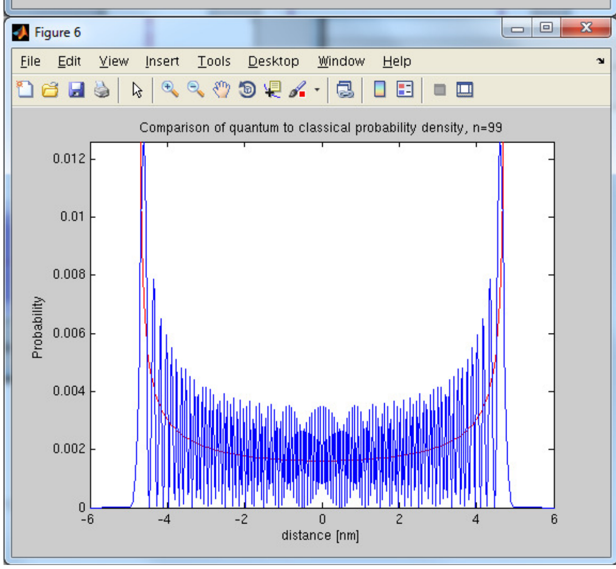
Figure 6
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Comparison of quantum to classical probability density, n=99
Probability
distance [nm]

## Problem 2.14

The new allowed energies are $E'_n = (n + \frac{1}{2})\hbar\omega' = 2(n + \frac{1}{2})\hbar\omega = \hbar\omega, 3\hbar\omega, 5\hbar\omega, \ldots$. So the probability of getting $\frac{1}{2}\hbar\omega$ is $\boxed{\text{zero.}}$ The probability of getting $\hbar\omega$ (the new ground state energy) is $P_0 = |c_0|^2$, where $c_0 = \int \Psi(x,0)\psi'_0\, dx$, with

$$\Psi(x,0) = \psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}, \quad \psi_0(x)' = \left(\frac{m2\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m2\omega}{2\hbar}x^2}.$$

So

$$c_0 = 2^{1/4}\sqrt{\frac{m\omega}{\pi\hbar}} \int_{-\infty}^{\infty} e^{-\frac{3m\omega}{2\hbar}x^2}\, dx = 2^{1/4}\sqrt{\frac{m\omega}{\pi\hbar}} 2\sqrt{\pi}\left(\frac{1}{2}\sqrt{\frac{2\hbar}{3m\omega}}\right) = 2^{1/4}\sqrt{\frac{2}{3}}.$$

Therefore

$$P_0 = \boxed{\frac{2}{3}\sqrt{2} = 0.9428.}$$

## Problem 2.15

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\xi^2/2}, \text{ so } P = 2\sqrt{\frac{m\omega}{\pi\hbar}} \int_{x_0}^{\infty} e^{-\xi^2} dx = 2\sqrt{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{\hbar}{m\omega}} \int_{\xi_0}^{\infty} e^{-\xi^2} d\xi.$$

Classically allowed region extends out to: $\frac{1}{2}m\omega^2 x_0^2 = E_0 = \frac{1}{2}\hbar\omega$, or $x_0 = \sqrt{\frac{\hbar}{m\omega}}$, so $\xi_0 = 1$.

$$P = \frac{2}{\sqrt{\pi}} \int_1^{\infty} e^{-\xi^2} d\xi = 2(1 - F(\sqrt{2})) \text{ (in notation of CRC Table)} = \boxed{0.157.}$$