1. Implement the finite-differences approximation: write a MATLAB script to solve the problem of an electron in a triangular well,

$$V(x)=\alpha x \text{ for } x>0, \text{ infinite otherwise, where } \alpha=10^9 \text{ eV/cm.}$$

Plot the probability density $\Psi_n^*\Psi_n$ for n=1,2,3 and the energy spectrum E_n for n<10. [Hint: Use a spatial discretization of ~10^{-9} cm.] For n=1, evaluate $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$, and Δx and Δp to confirm Heisenberg's uncertainty principle.

2. A particle in an infinite square well has an initial wave function that is an equal superposition of the two first states:

$$\Psi(x, 0) = A(\Psi_1(x) + \Psi_2(x)). \qquad (1)$$

i) Normalize $\Psi(x, 0)$.
ii) Find $\Psi(x, t)$ and $|\Psi(x, t)|^2$.
iii) Compute $\langle x\rangle$ and notice it is oscillatory. What is the amplitude and angular frequency of this oscillation?

Although the overall phase of the wave function is of no physical significance (it cancels out whenever you calculate a measurable quantity), the relative phase of the coefficients in eq. (1) does matter. For example, suppose we take

$$\Psi(x, 0) = A(\Psi_1(x) + e^{i\phi}\Psi_2(x))$$

instead of eq. (1). Find
iv) $\Psi(x, t)$,
v) $|\Psi(x, t)|^2$ and
vi) $\langle x\rangle$ and compare with the $\phi = 0$ case.

3. Suppose a particle in a box (infinite square well) has a triangular initial state wavefunction:

$$\psi(x) = \begin{cases} A\left(x+\frac{L}{2}\right), & -L/2 < x < 0 \\ A\left(\frac{L}{2}-x\right), & 0 < x < L/2 \\ 0, & \textit{otherwise} \end{cases}$$

As you can see from the wavefunction, we have chosen to measure the *x* coordinate from the center of the box for this problem. This makes the mathematics simpler. In this coordinate system, the stationary states appear as

$$\varphi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\cos\left(\frac{n\pi x}{L}\right), & n = odd \\ \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right), & n = even \end{cases}$$

a) What are the units of the normalization constant *A*?
b) Calculate *A* in terms of *L*.
c) Expand the wavefunction $\psi(x)$ in terms of the stationary states, and calculate the expansion coefficients $\{a_n\}$.
d) If the particle is an electron, and it is trapped in a conductor which is one nanometer long, calculate the probability that an energy measurement leaves the electron in the n = 3 state, and calculate the energy of that state in eV.