

1. Using the Fourier Transform, find the solution $f(x,t)$ of the “drift-diffusion equation” subject to initial conditions $f(x,t=0)=\delta(x)$:

$$\frac{df}{dt} = D \frac{d^2f}{dx^2} - v \frac{df}{dx}$$

Where D (diffusion coefficient) and v (velocity) are positive constants. Interpret the result.

2. Hermitian matrices:

- Write a MATLAB/octave **function** which returns a random, complex-valued **hermitian** matrix of arbitrary size N .
- Demonstrate that the eigenvalues are real by executing the function several times and evaluating the imaginary part (for example, using `imag()`).
- Demonstrate that the eigenvectors form a complete orthonormal set of basis functions (for instance, by calculating the inner product between several of them.)

3. Using separation of variables and the method of finite differences, convert the (differential) classical wave equation

$$\frac{d^2f}{dx^2} = \frac{1}{c^2} \frac{d^2f}{dt^2}$$

with fixed boundary conditions $f(x=0,t)=f(x=L,t)=0$ (which corresponds to a “wave on a string” between two rigid walls) into a matrix eigenvalue problem for the spatial modes.

- Use MATLAB/octave to solve for the eigenvalues and plot the ten lowest. Compare to the analytic result.
- Plot the **three** eigenvectors associated with the **three** eigenvalues having smallest absolute value. Compare to the analytic result.
- By calculating differences with the analytic wavenumber $k=n\pi/L$, show what happens to the accuracy of this approximation to eigenvalues as the size of the matrix (and the finite difference Δx) changes.

Your eigenvector plot should look something like this (for $N=10$):

