PHYS401 HW1: due 3PM Monday Feb. 6

Complex Numbers:

1.Simplify this number: i^i . Is it real/imaginary/complex?
2.Write this number in complex Cartesian form (real plus imaginary parts): $-1^{1/3}$. Derive a general expression for fractional powers of -1 (i.e. $-1^{1/n}$) in this form.
3.Use Euler's formula to prove the following trig identities

$$\sin^2(u) = \frac{1-\cos(2u)}{2}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

4.Prove the following identities for complex numbers:
- a) Re(z) = (z +z*)/2
- b) Im(z) = (z - z*)/2i
- c) cos(z) = [exp(iz) + exp(-iz)]/2
- d) sin(z) = [exp(iz) - exp(-iz)]/2i

Fourier Transforms:

1. Prove the "Modulation theorem", i.e.:
If the Fourier Transform of *F(x)* is *A(k)*, then the Fourier Transform of $F(x)\cos(k_0x)$ is $\frac{1}{2}[A(k+k_0)+A(k-k_0)]$

2.
a) Show that any function can be written as the sum of an even function and an odd function.
b) Show that the following is true:

$$F(x) = \frac{1}{\sqrt{\pi}}\int_0^\infty C(k)\cos kx\,dk + \frac{1}{\sqrt{\pi}}\int_0^\infty S(k)\sin kx\,dk$$

Find C(k) and S(k) in terms of $A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(x)e^{-ikx}dx$.

3. Symmetry
a) Find a general relationship between *A(k)* and *A(-k)* so that *F(x)* will be purely real.
b) Find a general relationship between *A(k)* and *A(-k)* so that *F(x)* will be purely imaginary.

Differential equations

1. Derive the scalar wave equation for one component of the vector electric field $E_x(z,t)$ from Maxwell's equations [Assume $\vec{E} = E_x(z,t)\hat{x}$].

2. Use the Fourier transform to derive the "d'Alembert" solution $E_x(z \pm ct)$ of the wave equation.

3. Solve for the ***general*** solution to the following linear, ordinary differential equations. (*A* is a positive constant.)

(a) $\dfrac{df}{dz} = Af$

(b) $\dfrac{d^2 f}{dr^2} = Af$

(c) $\dfrac{d^2 f}{dt^2} = -Af$

(d) $\dfrac{d^2 f}{dy^2} = 0$

(e) $\dfrac{d^2 f}{dx^2} = A$