

Consider the functions  $\sin \theta$  and  $\cos \theta$

What are their “lengths” as vectors in the IP space of functions?

$$\|\sin\|^2 = \int_0^{2\pi} \sin^2 \theta \, d\theta$$

$$\|\cos\|^2 = \int_0^{2\pi} \cos^2 \theta \, d\theta$$

Carry out these integrals in two ways:

(a) By drawing careful sketches of the graphs of the two functions and thinking about what the integral means.

(b) By expressing  $\sin$  and  $\cos$  in terms of complex exponentials and evaluating

$$\int_0^{2\pi} e^{ni\theta} \, d\theta$$

for an arbitrary value of  $n$ .

(Do you have to do something special for  $n = 0$ ?)