We have shown that the set of all functions of the form

$$f(\theta) = a\sin\theta + b\cos\theta$$

where a and b are arbitrary complex numbers form a linear space.

Do the functions of the form

$$f(\theta) = a_1\sin\theta + a_2\sin 2\theta + a_3\sin 3\theta + \ldots$$

$$= \sum_{n=1}^{\infty} a_n \sin n\theta$$

where a_i are arbitrary complex numbers form a linear space?