Consider whether functions of the form

$$f(\theta) = a_1 \sin\theta + a_2 \sin 2\theta + a_3 \sin 3\theta + \ldots$$

$$= \sum_{n=1}^{\infty} a_n \sin n\theta$$

where a_i are arbitrary complex numbers form an inner product space.

If we define

$$I(f,g) = \int_0^{2\pi} f^*(\theta)\, g(\theta)\, d\theta$$

does our set of functions form an IP space?