

An arbitrary continuous function, $f(x)$, that satisfies $f(0) = f(L) = 0$ can be expanded in the basis functions,

$$e_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$f(x) = \sum_{n=1}^{\infty} f_n e_n \quad \text{or} \quad |f\rangle = \sum_{n=1}^{\infty} f_n |e_n\rangle$$

If you are given f , use the orthonormality of the basis to derive a calculable expression for the constants (coordinates) f_n .