Analytic Functions

(a) What are the Cauchy-Riemann conditions for a function

$$f(x,y) = u(x,y) + iv(x,y)$$

to be analytic (*u* and *v* are real functions).

(b) Which of these functions are analytic?

$$f(x,y) = x^2 + y^2$$
$$f(x,y) = x^2 - y^2$$
$$f(x,y) = x^2 - y^2 - 2ixy$$
$$f(x,y) = x^2 + y^2 + 2ixy$$
$$f(x,y) = x^2 - y^2 + 2ixy$$

(c) Does the chain rule

$$\frac{d}{dz} f(g(z)) = \frac{df}{dg}\frac{dg}{dz}$$

hold for analytic functions?