The rules which apply to the familiar Cartesian linear space may be generalized to a more exotic function vector space, such as the space of sines (sin(*nx*)). In both spaces, any vector in the space may be constructed from a set of basis vectors. In the 3-D Cartesian position vector space, these are typically represented by $|e_1\rangle$, $|e_2\rangle$, and $|e_3\rangle$ which can be any orthonormal set of vectors which span the space, but commonly $\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}$. The set of sines has an analogous set of basis vectors, $|e_n\rangle = \frac{1}{\sqrt{\pi}}\sin(n\theta)$. This is an infinite dimensional basis (*n* can be any integer!), but again, any function can be represented as these basis vectors. [*better: as a sum over these basis vectors*]

Orthonormality isn't necessary in constructing a basis, but it is convenient. Orthonormality is the condition that, for some vectors [better: in the basis] $|e_n\rangle$ and $|e_m\rangle$, $\langle e_n|e_m\rangle = \delta_{nm}$. In Cartesian space, $|e_1\rangle$ and $|e_2\rangle$ are obviously perpendicular in the above basis and $\langle e_1|e_1\rangle$ is, again, clearly 1.

When generalizing to a function space, though, the inner product rules become more complicated. $\langle e_n|e_m\rangle$ is now equivalent to

$$\int_0^{2\pi} dx \left(\frac{1}{\sqrt{\pi}}\sin(nx)\right)^* \left(\frac{1}{\sqrt{\pi}}\sin(mx)\right) = \begin{cases}1 & \text{if } n = m\\ 0 & \text{if } n \neq m\end{cases}$$

and the normalization condition is

$$\int_0^{2\pi} dx\, |A_n \sin(nx)|^2 = 1$$

where A_n can be solved for to equal $1/\sqrt{\pi}$.

In Cartesian space, completeness is the concept that any vector in the space may be represented by a sum of the basis vectors times some constants:

$$|v\rangle = \sum_{n=1}^{3} a_n |e_n\rangle,$$

similarly, in function space, a vector

$$|v\rangle = \sum_{n=1}^{\infty} b_n |e_n\rangle;$$

the only difference is the summation, to three versus to infinity, and of course the different bases.

Function space is remarkably similar to Cartesian 3-D space, except that some operations must be redefined, such as the inner product, to make sense. Dirac notation emphasizes the similarities: the inner product is still written the same way. However, when evaluating it, the difference between the two spaces become more pronounced.