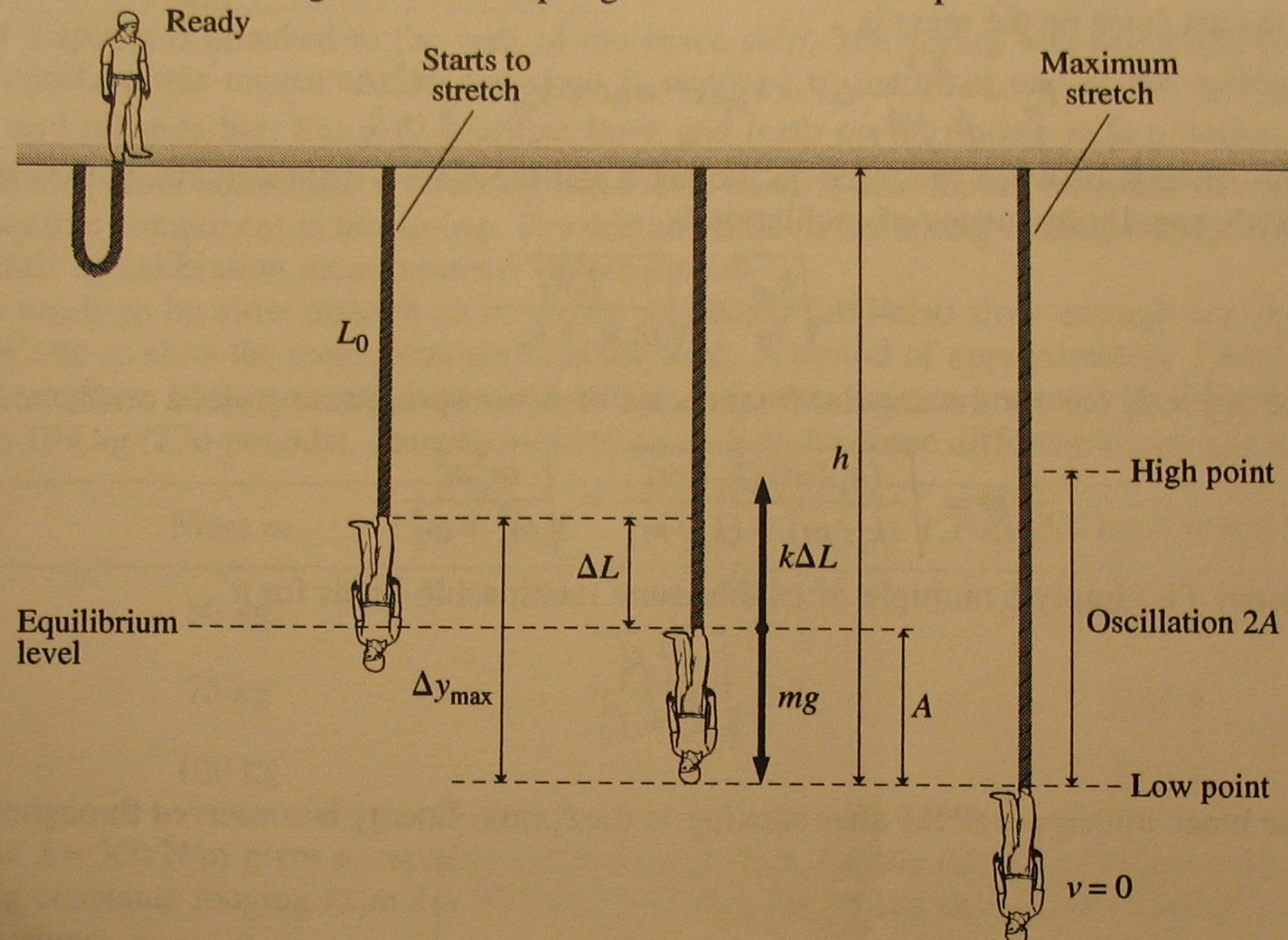
14.78. Model: Model the bungee cord as a spring. The motion is damped SHM.

Visualize:



Solve: **(a)** For light damping, the oscillation period is

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = m\left(\frac{2\pi}{T}\right)^2 = (75\text{ kg})\left(\frac{2\pi}{4.0\text{ s}}\right)^2 = 185\text{ N/m}$$

(b) The maximum speed is $v_{max} = \omega A = (2\pi/4.0\text{ s})(11.0\text{ m}) = 17.3\text{ m/s}$.

(c) Jose oscillates about the equilibrium position at which he would hang at rest. Balancing the forces, $\Delta L = mg/k = (75\text{ kg})(9.80\text{ m/s}^2)/(185\text{ N/m}) = 3.97\text{ m}$. Jose's lowest point is 11.0 m below this point, so the bungee cord is stretched by $\Delta y_{max} = \Delta L + A = 14.97\text{ m}$. Choose this lowest point as $y = 0$. Because Jose is instantaneously at rest at this point, his energy is entirely the elastic potential energy of the stretched bungee cord. Initially, his energy was entirely gravitational potential energy. Equating his initial energy to his energy at the lowest point,

$$U_{\text{lowest point}} = U_{\text{highest point}} \Rightarrow \tfrac{1}{2}k(\Delta y_{max})^2 = mgh$$

$$h = \frac{k(\Delta y_{max})^2}{2mg} = \frac{(185\text{ N/m})(14.97\text{ m})^2}{2(75\text{ kg})(9.80\text{ m/s}^2)} = 28.2\text{ m}$$

Jose jumped 28.2 m above the lowest point.

(d) The amplitude decreases due to damping as $A(t) = Ae^{-bt/2m}$. At the time when the amplitude has decreased from 11.0 m to 2.0 m,

$$\frac{2.0\text{ m}}{11.0\text{ m}} = e^{-bt/2m} \Rightarrow t = -\frac{2m}{b}\ln\left(\frac{2}{11}\right) = -\frac{2(75\text{ kg})}{6.0\text{ kg/s}}(-1.705) = 42.6\text{ s}$$

With a period of 4.0 s, the number of oscillations is $N_{osc} = (42.6\text{ s})/(4.0\text{ s}) = 10.7$ oscillations.

Good Vibes: Introduction to Oscillations

Description: Several conceptual and qualitative questions related to main characteristics of simple harmonic motion: amplitude, displacement, period, frequency, angular frequency, etc. Both graphs and equations are used.
Learning Goal: To learn the basic terminology and relationships among the main characteristics of simple harmonic motion.

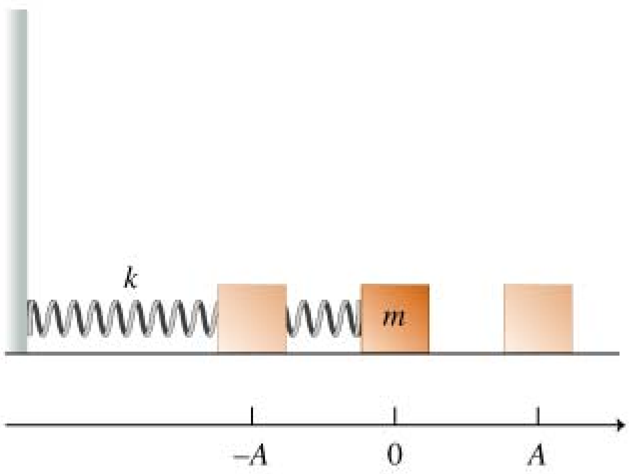
Motion that repeats itself over and over is called *periodic motion*. There are many examples of periodic motion: the earth revolving around the sun, an elastic ball bouncing up and down, or a block attached to a spring oscillating back and forth.

The last example differs from the first two, in that it represents a special kind of periodic motion called *simple harmonic motion*. The conditions that lead to simple harmonic motion are as follows:

- There must be a position of *stable equilibrium*.
- There must be a restoring force acting on the oscillating object. The direction of this force must always point toward the equilibrium, and its magnitude must be directly proportional to the magnitude of the object's displacement from its equilibrium position. Mathematically, the restoring force $\vec{F}$ is given by $\vec{F} = -k\vec{x}$, where $\vec{x}$ is the displacement from equilibrium and k is a constant that depends on the properties of the oscillating system.
- The resistive forces in the system must be reasonably small.

In this problem, we will introduce some of the basic quantities that describe oscillations and the relationships among them.

Consider a block of mass m attached to a spring with force constant k, as shown in the figure



. The spring can be either stretched or compressed. The block slides on a frictionless horizontal surface, as shown. When the spring is relaxed, the block is located at $x = 0$. If the block is pulled to the right a distance A and then released, A will be the *amplitude* of the resulting oscillations.

Assume that the mechanical energy of the block-spring system remains unchanged in the subsequent motion of the block.

Part A

After the block is released from $x = A$, it will

ANSWER:

- ☐ remain at rest.
- ☐ move to the left until it reaches equilibrium and stop there.
- ☐ move to the left until it reaches $x = -A$ and stop there.
- ☑ move to the left until it reaches $x = -A$ and then begin to move to the right.

As the block begins its motion to the left, it accelerates. Although the restoring force decreases as the block approaches equilibrium, it still pulls the block to the left, so by the time the equilibrium position is reached, the block has gained some speed. It will, therefore, pass the equilibrium position and keep moving, compressing the spring. The spring will now be pushing the block to the right, and the block will slow down, temporarily coming to rest at $x = -A$.

After $x = -A$ is reached, the block will begin its motion to the right, pushed by the spring. The block will pass the equilibrium position and continue until it reaches $x = A$, completing *one cycle* of motion. The motion will then repeat; if, as we've assumed, there is no friction, the motion will repeat indefinitely.

The time it takes the block to complete one cycle is called the *period*. Usually, the period is denoted T and is measured in seconds.

The *frequency*, denoted f, is the number of cycles that are completed per unit of time: $f = 1/T$. In SI units, f is measured in inverse seconds, or hertz (Hz).

Part B

If the period is doubled, the frequency is

ANSWER:

- ☐ unchanged.
- ☐ doubled.
- ☑ halved.

Part C

An oscillating object takes 0.10 s to complete one cycle; that is, its period is 0.10 s.
What is its frequency f?
Express your answer in hertz.

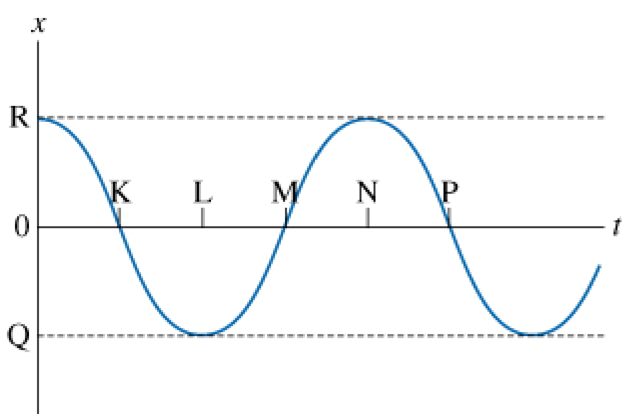
ANSWER: f = 10 Hz

Part D

If the frequency is 40 Hz, what is the period T?
Express your answer in seconds.

ANSWER: T = 0.025 s

The following questions refer to the figure



that graphically depicts the oscillations of the block on the spring.

Note that the vertical axis represents the *x* coordinate of the oscillating object, and the horizontal axis represents time.

Part E

Which points on the *x* axis are located a distance A from the equilibrium position?

ANSWER:
- R only
- Q only
- both R and Q

Part F

Suppose that the period is T. Which of the following points on the *t* axis are separated by the time interval T?

ANSWER:
- ☐ K and L
- ☐ K and M
- ☒ K and P
- ☐ L and N
- ☐ M and P

K and P are separated by phase interval of 2π, or the block gets back to the same point.

Now assume that the *x* coordinate of point R is 0.12 m and the *t* coordinate of point K is 0.0050 s.

Part G

What is the period T?

Hint G.1 How to approach the problem

In moving from the point $t=0$ to the point K, what fraction of a full wavelength is covered? Call that fraction a. Then you can set $aT = 0.005\ \mathrm{s}$. Dividing by the fraction a will give the period T.

Express your answer in seconds.

ANSWER: T = 0.02 s

Part H

How much time t does the block take to travel from the point of maximum displacement to the opposite point of maximum displacement?
Express your answer in seconds.

ANSWER: t = 0.01 s

The block travels only half the period.

Part I

What distance d does the object cover during one period of oscillation?
Express your answer in meters.

ANSWER: d = 0.48 m

4 times of the amplitude.

Part J

What distance d does the object cover between the moments labeled K and N on the graph?

Express your answer in meters.

ANSWER: d = 0.30 m

3 times of the amplitude.

Energy of a Spring

Description: Short quantitative problem relating the total, potential, and kinetic energies of a mass that is attached to a spring and undergoing simple harmonic motion. This problem is based on Young/Geller Quantitative Analysis 11.1

An object of mass m attached to a spring of force constant k oscillates with simple harmonic motion. The maximum displacement from equilibrium is A and the total mechanical energy of the system is E.

Part A

What is the system's potential energy when its kinetic energy is equal to $\frac{3}{4}E$?

Hint A.1 How to approach the problem

Since the sum of kinetic and potential energies of the system is equal to the system's total energy, if you know the fraction of total energy corresponding to kinetic energy you can calculate how much energy is potential energy. Moreover, using conservation of energy you can calculate the system's total energy in terms of the given quantities k and A. At this point you simply need to combine those results to find the potential energy of the system in terms of k and A.

Part A.2 Find the fraction of total energy that is potential energy

When the kinetic energy of the system is equal to $\frac{3}{4}E$, what fraction of the total energy E is potential energy?

Hint A.2.a Conservation of mechanical energy

In a system where no forces other than gravitational and elastic forces do work, the sum of kinetic energy K and potential energy U is conserved. That is, the total energy E of the system, given by $E = K + U$, is constant.

Express your answer numerically.

ANSWER: $\frac{1}{4}$

Part A.3 Find the total energy of the system

What is the total mechanical energy of the system, E?

Hint A.3.a How to approach the problem

If you apply conservation of energy to the system when the object reaches its maximum displacement, you can calculate the system's total energy E in terms of the given quantities A and k. In fact, when the object is at its maximum displacement from equilibrium, its speed is momentarily zero and so is its kinetic energy. It follows that the system's energy at this point is entirely potential, that is, $E = U$, where U is the spring's elastic potential energy.

Hint A.3.b Elastic potential energy

The elastic potential energy U of a spring that has been compressed or stretched by a distance x is given by

$$U = \frac{1}{2}kx^2,$$

where k is the force constant of the spring.

Express your answer in terms of some or all of the variables m, k, and A.

ANSWER: $E = \frac{kA^2}{2}$

ANSWER:

- ○ kA^2
- ○ $\frac{kA^2}{2}$
- ○ $\frac{kA^2}{4}$
- ◉ $\frac{kA^2}{8}$

Part B

What is the object's velocity when its potential energy is $\frac{2}{3}E$?

Hint B.1 How to approach the problem

You can calculate the object's velocity using energy considerations. Calculate the fraction of the system's total energy that is kinetic energy and then find the object's velocity from the definition of kinetic energy. To simplify your expression write the total energy in terms of k and A. Alternatively, you could directly use the formula for the object's velocity in terms of the variables k, m, A, and displacement x derived from energy considerations. The only unknown quantity in such a formula would be the object's displacement x, which can be calculated from the system's potential energy.

Part B.2 Find the kinetic energy

If the system's potential energy is $\frac{2}{3}E$, what is the system's kinetic energy?

Hint B.2.a Conservation of mechanical energy

In a system where no forces other than gravitational and elastic forces do work, the sum of kinetic energy K and potential energy U is conserved. That is, the total energy E of the system, given by $E = K + U$, is constant.

Hint B.2.b Total energy of the system

The total energy of a system consisting of an object attached to a horizontal spring of force constant k is given by

$$E = \frac{1}{2}kA^2,$$

where A is the maximum displacement of the object from its equilibrium position.

ANSWER:

- ○ $\frac{1}{2}kA^2$
- ○ $\frac{1}{3}kA^2$
- ◉ $\frac{1}{6}kA^2$
- ○ $\frac{2}{3}kA^2$

Now use your result and the definition of kinetic energy as $\frac{1}{2}mv^2$ to find the object's speed v.

Hint B.3 Formula for the velocity in terms of position

The velocity of an object of mass m undergoing simple harmonic motion is given by

$$v = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2},$$

where k is the force constant of the system, x is the object's position, and A is maximum displacement.

Part B.4 Find the object's position

When the system's potential energy is $\frac{2}{3}E$, what is the displacement x of the object from its equilibrium position?

Hint B.4.a Elastic potential energy

The elastic potential energy U of a spring that has been compressed or stretched by a distance x is given by

$$U = \frac{1}{2}kx^2,$$

where k is the force constant of the spring.

Hint B.4.b Total energy of the system

The total energy of a system consisting of an object attached to a horizontal spring of force constant k is given by

$$E = \frac{1}{2}kA^2,$$

where A is the maximum displacement of the object from its equilibrium position.

ANSWER:

- ○ $\frac{2}{3}A$
- ◉ $\pm\sqrt{\frac{2}{3}}A$
- ○ $=\sqrt{\frac{2}{3k}}A$
- ○ $\pm\sqrt{\frac{1}{3}}A$

ANSWER:

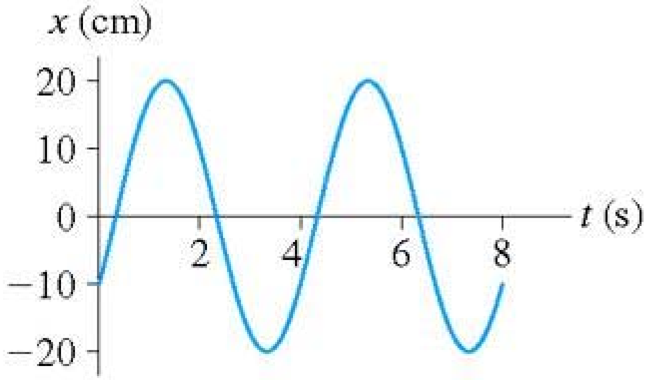
- ○ $\pm\sqrt{\frac{k}{m}}A$
- ○ $\pm\sqrt{\frac{k}{m}}\sqrt{\frac{2}{3}}A$
- ◉ $\pm\sqrt{\frac{k}{m}}\frac{A}{\sqrt{3}}$
- ○ $\pm\sqrt{\frac{k}{m}}\frac{A}{\sqrt{6}}$

Problem 14.6

Description: (a) What is the amplitude of the oscillation shown in the figure? (b) What is the frequency of this oscillation? (c) What is the phase constant?

Part A

What is the amplitude of the oscillation shown in the figure?



ANSWER: 20.0 cm

Part B

What is the frequency of this oscillation?

ANSWER: 0.250 Hz

Part C

What is the phase constant?
Express your answer in degrees.

ANSWER: − 120 °

240

Problem 14.16

Description: A 507 g mass oscillates with an amplitude of 10 cm on a spring whose spring constant is 20 N/m. At t =0 s the mass is 5.0 cm to the right of the equilibrium position and moving to the right. Determine: (a) The period. (b) The angular frequency. (...

Part A

The period. $\omega = \sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.507}{20}} = 1.00\,\text{s}.$

ANSWER: 1.00 s

Part B

The angular frequency. $\omega = \sqrt{\frac{k}{m}}$

ANSWER: 6.28 T rad/s

Part C

The phase constant. $\cos(\phi_{\circ}) = -\frac{5}{10} = \frac{1}{2}, \phi = \frac{\pi}{3} = 1.05 rad/s$

ANSWER: − 1.05 rad

Part D

The initial velocity. $v_{\circ} = \omega * r * \sin(\phi_{\circ}) = 6.28 * 0.10 * \frac{\sqrt{3}}{2} = 0.544 m/s$

ANSWER: 0.544 m/s

Part E

The maximum speed. $v_m = \omega * r = 6.28 * 0.10 = 0.628 m/s$

ANSWER: 0.628 m/s

Part F

The total energy. $E = \frac{1}{2} m v_m{}^2 = \frac{1}{2} * 0.507 * 0.628^2 = 0.100 J$

ANSWER: 0.100 J

Part G

The position at $t = 1.3$ s. $\phi = \phi_{\circ} + \omega t = -\frac{\pi}{3} + 2\pi * 1.3 = 48^{\circ}$

$x = r * \cos 48^{\circ} = 6.69 cm$

ANSWER: 6.72

6.69 cm

Part H

The velocity at $t = 1.3$ s. $v = -\omega * r * \sin 48^{\circ} = -6.28 * 0.1 * \sin 48^{\circ} = -0.467 m/s$

ANSWER: − 0.467 m/s

Problem 14.44

Description: The velocity of an object in simple harmonic motion is given by v_x(t)= - (0.35 m/s) sin(20t+ pi), where t is in s. (a) What is the first time after t =0 s at which the velocity is - 0.25 m/s? (b) What is the object's position at that time?

The velocity of an object in simple harmonic motion is given by $v_x(t) = -(0.35\ \mathrm{m/s})\sin(20t+\pi)$, where t is in s.

Part A

What is the first time after $t = 0\ \mathrm{s}$ at which the velocity is $-0.25\ \mathrm{m/s}$?

v_x(t)= - (0.35 m/s) sin(20t+ pi)=-0.25m/s

t=0.197s

ANSWER: 0.197 s

Part B

What is the object's position at that time? $\sin(20\mathrm{t}+\pi)=\frac{0.25}{0.35}=\frac{5}{7}$,

$v=\omega * r=0.35m/s$, $\omega=20rad/s$, therefore the position is

$x=r*\cos(20\mathrm{t}+\pi)=\frac{v}{\omega}*\frac{\sqrt{7^2-5^2}}{7}=1.22cm$.

ANSWER: 1.22 cm

Problem 14.57

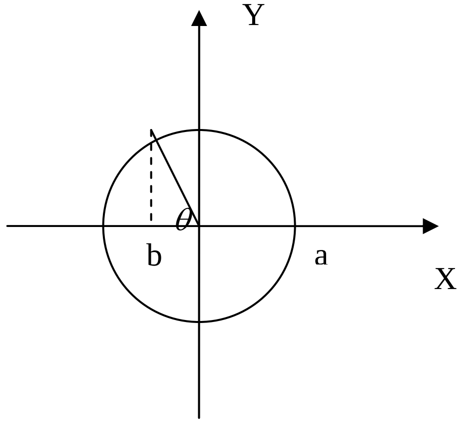
Description: A m mass on a l-m-long string is pulled a degree(s) to one side and released. (a) How long does it take for the pendulum to reach b degree(s) on the opposite side?

A 160 g mass on a 1.00-m-long string is pulled 7.10 $^\circ$ to one side and released.

Part A

How long does it take for the pendulum to reach 2.40 $^\circ$ on the opposite side?

ANSWER: $\left(\sqrt{\frac{l}{9.8}}\right)\mathrm{acos}\left(\frac{-b}{a}\right)$ s



Shown as the figure on the left. When the mass reaches to b degrees on the other side, the corresponding circular motion indicates that

$$t = \frac{\theta}{\omega} = \sqrt{\frac{l}{9.8}}\theta, where \theta = a\cos(-\frac{b}{a}).$$

Problem 14.60

Description:
In a science museum, a 110 kg brass pendulum bob swings at the end of a 15.0-m-long wire. The pendulum is started at exactly 8:00 a.m. every morning by pulling it 1.5 m to the side and releasing it. Because of its compact shape and smooth surface, the pendulum's damping constant is only 0.010 kg/s.

Part A

At exactly 12:00 noon, how many oscillations will the pendulum have completed?

ANSWER: 1852.5

The damping time constant is

m/b=11000s. $n = t\frac{\omega}{2\pi} = 4*3600*\sqrt{\frac{9.8}{15} - \frac{0.01^2}{4*110^2}} / 2\pi = 1852.5$

Part B

And what is its amplitude?

ANSWER: 0.780 m

$A(t) = A\exp(-t/2\tau) = 1.5*\exp(-t/2*11000)$

$Therefore, t = 4*60*60s, A(t) = 1.5*\exp(-4*60*60/2*11000) = 0.780m$

Problem 14.63

Description: A 0.980 kg (m) block is attached to a horizontal spring with spring constant 2200 N/m. The block is at rest on a frictionless surface. A 8.80 g(n) bullet is fired into the block, in the face opposite the spring, and sticks.

Part A

What was the bullet's speed if the subsequent oscillations have an amplitude of 12.6 cm?

ANSWER: $\frac{A\left(\sqrt{km+kn}\right)}{n}$ m/s

Conservation of momentum $nv_0 = (m+n)v$.

Conservation of energy $\frac{1}{2}(m+n)v^2 = \frac{1}{2}kA^2$.

$$v_0 = \frac{(m+n)}{n}v = \frac{m+n}{n}\sqrt{\frac{kA^2}{m+n}} = \frac{A}{n}\sqrt{k(m+n)}m/s.$$