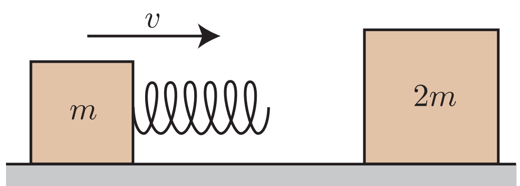
Physics 161 Sample Midterm 3

1. A block of mass m has a spring with spring constant k attached as shown below. The block is initially sliding on a horizontal frictionless surface with speed v. The block collides with a block of mass $2m$.



(a) (7 points) Is momentum conserved in the collision? Explain.

Take the system to consist of both masses. Since there is no friction, there is no net external force on the system. Therefore, the total momentum of the system is conserved.

(b) (7 points) Is energy conserved in the collision? Explain.

The only forces acting along the direction of motion are due to the spring. This is a conservative force ($W_{\text{spring}} = -\Delta U_{\text{spring}}$) so mechanical energy is conserved.

(c) (19 points) Find the maximum compression of the spring during the collision.

At the moment of maximum spring compression, the blocks have the same velocity v'. Take the $+x$ axis to point to the right. Conservation of momentum implies

$$mv = (3m)v'. \tag{1}$$

Conservation of energy implies

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(3m)v'^2 + \tfrac{1}{2}k\Delta s^2, \tag{2}$$

where Δs is the amount the spring is compressed. We can solve for v' using the first equation

$$v' = \tfrac{1}{3}v \tag{3}$$

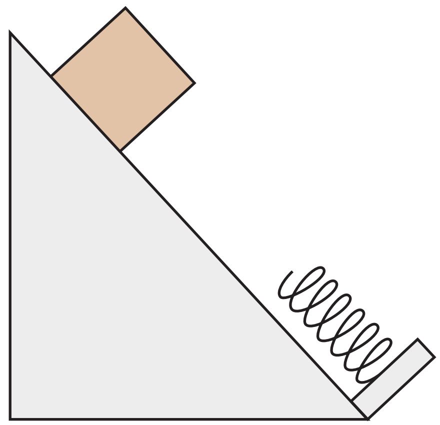
and substitute into the second equation to get

$$mv^2 = (3m)(\tfrac{1}{3}v)^2 + k\Delta s^2. \tag{4}$$

Solving for Δs gives

$$\Delta s = \sqrt{\frac{2mv^2}{3k}}. \tag{5}$$

2. (33 points) A block of mass 2.0 kg slides down an inclined plane that makes a 55° angle with the horizontal. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.33$. At the bottom of the incline, a spring with spring constant 120 N/m stops the downward slide of the block. Find the height from which the block must be released in order that the spring compresses by 0.10 m when the block is momentarily at rest.



The basic method is to use the conservation of energy

$$\Delta E = W_{\text{non-conservative}}. \tag{6}$$

The only non-conservative force is friction, so we must compute the work due to friction. Let h be the vertical height from which the block is released, measured from the point where the unstretched spring starts. Then the distance d along the plane that the block slides before coming to a halt is equal to the distance corresponding to h, plus the amount Δs that the spring is compressed:

$$d = \frac{h}{\sin\theta} + \Delta s, \tag{7}$$

where $\theta = 55°$ is the angle of the slope. The work done by friction is therefore

$$W_{\text{fric}} = -F_{\text{fric}} d = -\mu_k N d = -\mu_k (mg\cos\theta)\left(\frac{h}{\sin\theta} + \Delta s\right). \tag{8}$$

The initial and final energy is

$$E_i = mgh, \tag{9}$$

$$E_f = mg(-\Delta s \sin\theta) + \tfrac{1}{2}k\Delta s^2. \tag{10}$$

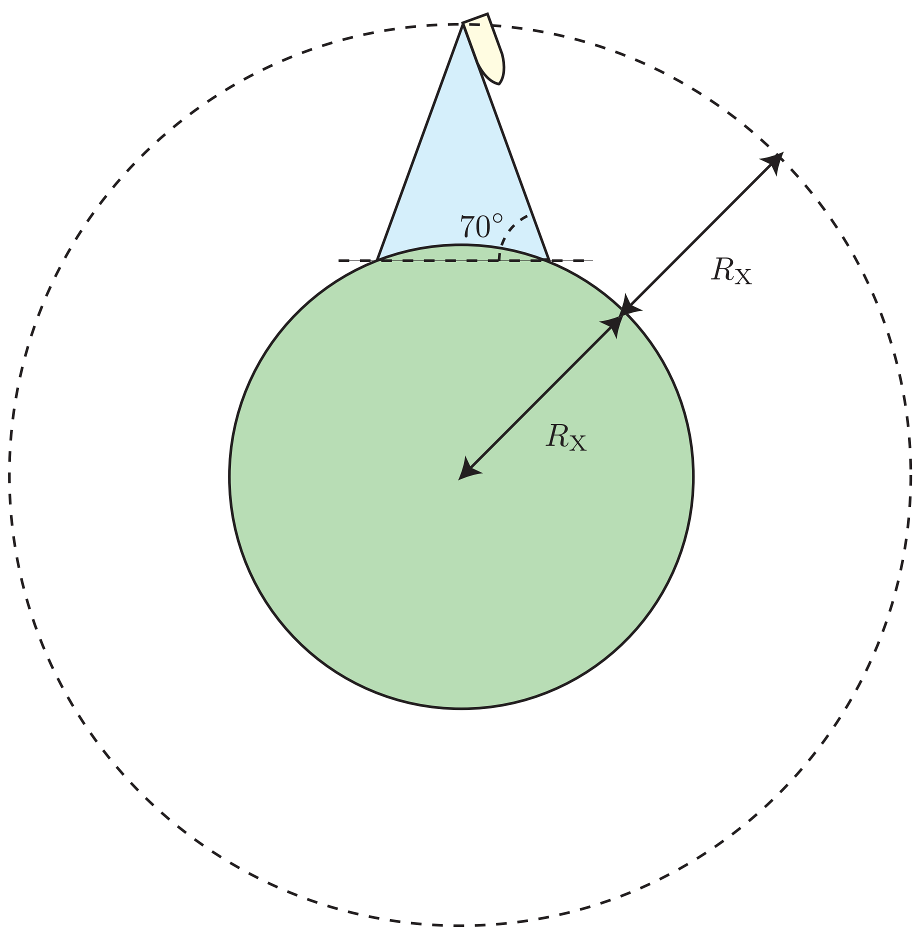
We therefore have

$$\Delta E = mg(h + \Delta s \sin\theta) - \tfrac{1}{2}k\Delta s^2 = -\mu_k(mg\cos\theta)\left(\frac{h}{\sin\theta} + \Delta s\right). \qquad (11)$$

This is a fairly complicated looking formula, but it is linear in h. Solving, we get

$$h = \frac{\frac{1}{2}k\Delta s^2 - mg(\sin\theta + \mu_k\cos\theta)\Delta s}{mg(1 + \mu_k\cot\theta)}. \qquad (12)$$

3. (33 points) Planet X has surface gravity $g_X = 3.5$ m/s^2. It has a giant ice-covered volcano whose vertical height is equal to the radius of the planet $R_X = 5.3 \times 10^6$ m, and whose slope is 70°. A space sled slides from the top of the mountain. Find its speed at the bottom if we neglect air resistance and friction. (You do *not* need to know G or the mass of the planet in this problem.)



The only force along the direction of motion is gravity, so we can use conservation of energy:

$$E_i = E_f. \tag{13}$$

The initial energy is purely potential energy:

$$E_i = -\frac{GM_X m}{2R_X}, \tag{14}$$

while the final energy is gravitational plus kinetic:

$$E_f = -\frac{GM_X m}{R_X} + \tfrac{1}{2}mv^2. \tag{15}$$

Note that the gravitational potential energy depends only on the distance from the center of the planet. Setting these equal to each other, we see that the mass m of the sled cancels:

$$-\frac{GM_X}{2R_X} = -\frac{GM_X}{R_X} + \tfrac{1}{2}v^2. \tag{16}$$

We do not know the mass of the planet, but we do know the surface gravity. A mass m at the surface of the planet falling under the influence of gravity has an acceleration

$$a = \frac{F_{\text{grav}}}{m} = \frac{1}{m}\frac{GMm}{R_X^2}, \tag{17}$$

so we identify

$$g_X = \frac{GM}{R_X^2}. \tag{18}$$

We can use this to rewrite Eq. (16) as

$$-\frac{g_X R_X}{2} = -\frac{g_X R_X}{+}\tfrac{1}{2}v^2. \tag{19}$$

Solving for v gives

$$v = \sqrt{g_X R_X}. \tag{20}$$