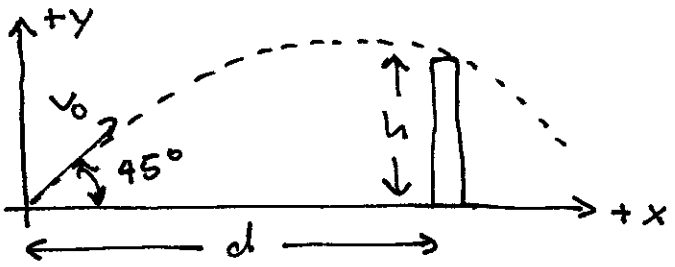
Sample Exam 2

Problem 1



$$x = v_0 \cos 45^\circ \cdot t$$
$$= \frac{v_0}{\sqrt{2}} t$$
$$y = v_0 \sin 45^\circ t - \frac{1}{2} g t^2$$
$$= \frac{v_0}{\sqrt{2}} t - \frac{1}{2} g t^2$$

Cannoball grazes top of wall: $x = d$, $y = h$

$$d = \frac{v_0}{\sqrt{2}} t$$
$$h = \frac{v_0}{\sqrt{2}} t - \frac{1}{2} g t^2$$

Elim t:

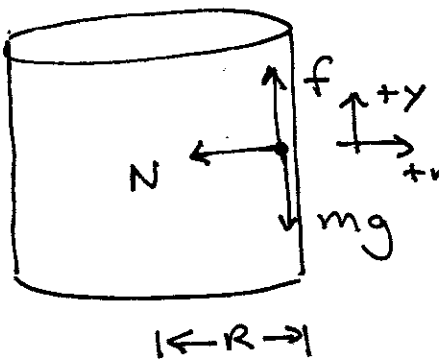
$$t = \frac{\sqrt{2}}{v_0} d$$
$$\Rightarrow h = \frac{v_0}{\sqrt{2}} \left(\frac{\sqrt{2}}{v_0} d \right) - \frac{1}{2} g \left(\frac{\sqrt{2}}{v_0} d \right)^2$$
$$= d - \frac{1}{2} g \frac{2 d^2}{v_0^2}$$

Solve for v_0:

$$h - d = -g \frac{d^2}{v_0^2}$$
$$\frac{1}{h-d} = -\frac{v_0^2}{g d^2}$$
$$v_0^2 = -\frac{g d^2}{h-d}$$
$$\boxed{v_0 = d \left(\frac{g}{d-h} \right)^{1/2}} = 11.8 \text{ m/s}$$

Problem 2

sys = girl



(a) N = normal force
= force of wall on girl
⊥ to wall

f = friction
= force of wall on girl
∥ to wall

mg = weight of girl
= force of gravity on girl

(b) N2: $F_{net,y} = f - mg = 0$ $(a_y = 0)$

$\Rightarrow \boxed{f = mg}$

$F_{net,r} = -N = ma_r$

a_r = inward accel $= -\frac{v^2}{R}$

Relate v to ω = revolutions/sec

$\omega = \frac{1}{T}$, T = time for revolution $= \frac{2\pi R}{v}$

$\Rightarrow \omega = \frac{v}{2\pi R}$ or $v = 2\pi R\omega$

$a_r = -\frac{1}{R}(2\pi R\omega)^2 = -4\pi^2 R\omega^2$

Plug into N2: $\boxed{N = m \cdot 4\pi^2 R\omega^2}$

Min coeff of static friction

$\Rightarrow f = ma_x = \mu_s N$

Elim f:

$$\mu_s N = mg$$

$$N = m \cdot 4\pi^2 R\omega^2$$

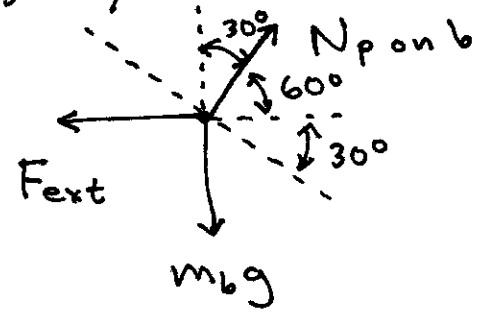
Elim N:

$$\mu_s \cdot \cancel{m} \cdot 4\pi^2 R\omega^2 = \cancel{m} g$$

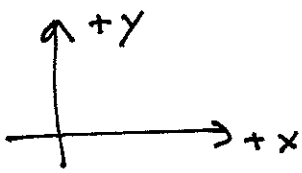
$$\boxed{\mu_s = \frac{g}{4\pi^2 R\omega^2}} = 0.33$$

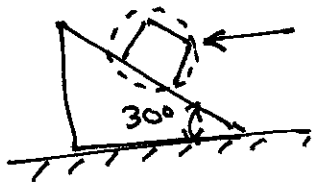
Problem 3

(a) sys = block

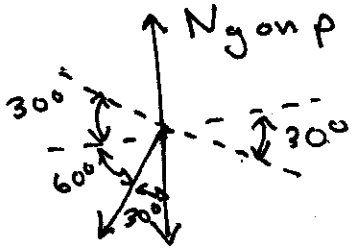


(b)

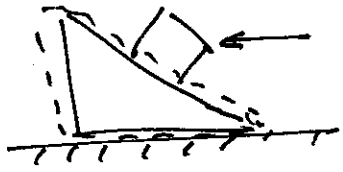




sys = plane (p)



g = ground



(b) N2 for block:

$$F_{net,x} = \boxed{-F_{ext} + N_{p\,on\,b}\sin 30^\circ = m_b a_{bx}}$$

$$F_{net,y} = \boxed{N_{p\,on\,b}\cos 30^\circ - m_b g = 0} \quad (a_{by} = 0)$$

N2 for plane:

$$F_{net,x} = \boxed{-N_{b\,on\,p}\sin 30^\circ = m_p a_{px}}$$

$$F_{net,y} = \boxed{N_{g\,on\,p} - N_{b\,on\,p}\cos 30^\circ - m_p g = 0} \quad (a_{py}=0$$

Accel constraint

$$a_{bx} = a_{px} = a_x$$

N3: $N_{p\,on\,b} = N_{b\,on\,p} = "N"$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 30^\circ = \frac{1}{2}$

$$\Rightarrow \boxed{\begin{aligned} -F_{ext} + N\cdot\tfrac{1}{2} &= m_b a_x \\ N\cdot\tfrac{\sqrt{3}}{2} &= m_b g \\ -N\tfrac{1}{2} &= m_p a_x \\ N_{g\,on\,p} - N\cdot\tfrac{\sqrt{3}}{2} - m_p g &= 0 \end{aligned}}$$

Elim N; ignore last $\vec{F}$ equ

$$N = -2 m_p a_x$$

$$\Rightarrow \boxed{\begin{aligned} -F_{ext} + (-2 m_p a_x)\cdot\tfrac{1}{2} &= m_b a_x \\ (-2 m_p a_x)\cdot\tfrac{\sqrt{3}}{2} &= m_b g \end{aligned}}$$

Elim a_x:

$$a_x = -\frac{1}{\sqrt{3}\, m_p}\cdot m_b g$$

$$\Rightarrow -F_{ext} - m_p\left(-\frac{m_b g}{\sqrt{3}\, m_p}\right) = m_b\left(-\frac{m_b g}{\sqrt{3}\, m_p}\right)$$

$$-F_{ext} = -\frac{m_b g}{\sqrt{3}} - \frac{m_b^2 g}{\sqrt{3}\, m_p}$$

$$\boxed{F_{ext} = \frac{m_b(m_b+m_p)}{\sqrt{3}} g}$$

$$= 22.6\ N$$