FIELDS: GRAVITATIONAL, COULOMB $\vec{E},\vec{B}$

$\vec{G}$: A mass m located in a Gravitational field feels a force $\vec{F}_G = m\vec{G}_F$

Measure $\vec{F}_G$, map out $\vec{G}_F$.

A mass M located at the origin creates a $\vec{G}_F$

$$\vec{G}_F = -\frac{GM}{r^2}\hat{r}$$

consequently, $\Sigma_c \vec{G}_F \bullet \vec{\Delta A} = -4\pi G \Sigma M_i$

FLUX OF $\vec{G}_F$ Through a closed surface is determined solely by the masses enclosed by the surface.

$\vec{E}$: A charge q located in an $\vec{E}$-field feels a force $\vec{F}_E = q\vec{E}$.

Measure $\vec{F}_E$, map out $\vec{E}$.

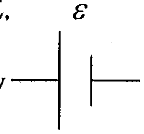
A stationary charge Q located at r=0 generates a coulomb $\vec{E}$-field

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

[+Q (source), -Q (sink)] consequently, $\Sigma_c \vec{E} \bullet \vec{\Delta A} = \frac{1}{\varepsilon_0}\Sigma Qi$

FLUX OF $\vec{E}$ THROUGH A CLOSED SURFACE IS DETERMINED SOLELY BY THE CHARGES ENCLOSED BY THE SURFACE.

ε

The Devices resulting from this are: (i) Battery 

(ii) Capacitor $C = \frac{Q}{V}$ which leads to energy density $\eta_E = \frac{1}{2}\varepsilon_0 E^2$

That is the energy contained in $1m^3$ vol. of $\vec{E}$-field

(iii) Resistor $R = \frac{V}{I}$ which leads to $\vec{J} = \sigma\vec{E}$, because $I = \vec{J} \bullet \vec{A}$

That is, if you apply $\vec{E}$ to a Conductor it responds by setting up a current density $\vec{J}$ whose magnitude is determined by the electrical conductivity σ.

To Summarize: if a stationary mass experiences a force when there is no visible agency applying the force, the mass must be located in a Gravitational, $\vec{G}$ field.

If a stationary charge experiences a force when there is no visible agency applying the force, the charge must be located in an Electric $\vec{E}$-field.

Next, we want to talk about a Magnetic Field: $\vec{B}$-field. The presence of a $\vec{B}$-field is revealed only if we have a charge q which is moving. Supposing there is a region of space where a uniform $\vec{B}$-field is present. If we take a charge q and give it a velocity $\vec{v}$ we will find the following feature of the force it experiences

i) The force is directly proportional to q and reverses with the sign of q.
ii) The force is directly proportional to v.
iii) It is always perpendicular to $\vec{v}$.
iv) It is directly proportional to B.
v) There is one direction of $\vec{v}$ where force is zero no matter how large $\vec{B}$ is.

All of these findings can be incorporated in a single expression for the force, namely

$$\vec{F}_B = q[\vec{v} \times \vec{B}]$$

And by definition of the vector product we know that magnitude of

$$F_B = qvB\sin(\vec{v}, \vec{B})$$

So this force is maximum when $\vec{v} \perp \vec{B}$ and zero if $\vec{v} \parallel \vec{B}$.

Also direction of $\vec{F}_B$ is given by the right-hand rule

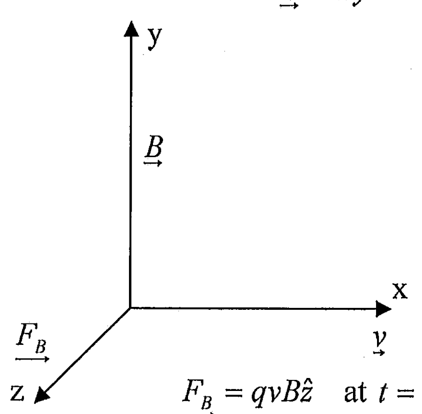
$q\vec{v} \parallel$ Thumb $\qquad \vec{B} \parallel$ Fingers $\qquad \vec{F}_B \perp$ Palm

The new field $\vec{B}$ has dimensions $MT^{-1}Q^{-1}$, the unit is TESLA and it is a VECTOR.

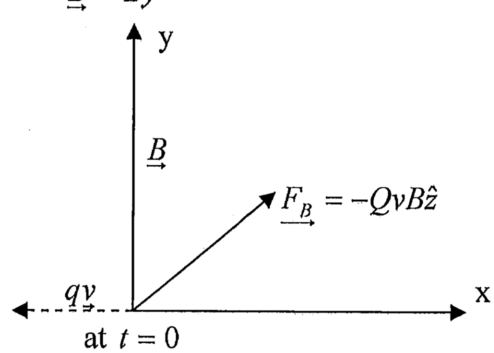
<u>Examples</u> $\qquad q$ positive $\qquad q$ negative

<u>$t = 0$</u> $\qquad \vec{v} = v\hat{x} \qquad t = 0 \quad \vec{v} = v\hat{x}$

$\vec{B} = B\hat{y} \qquad \vec{B} = B\hat{y}$

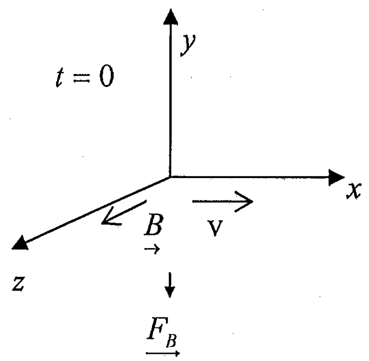


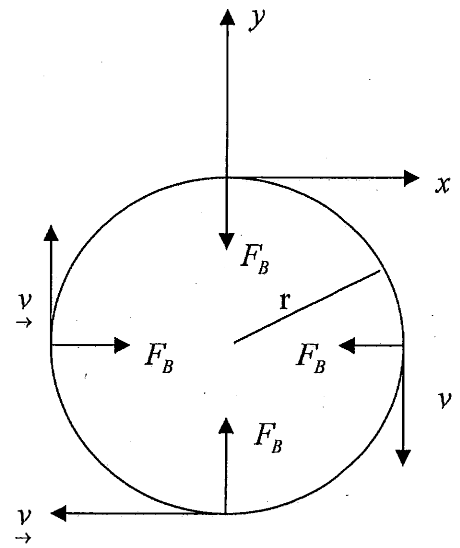
$\vec{F}_B = qvB\hat{z}$ at $t = 0$



So, if a moving charge experiences a force which is always perpendicular to its velocity and there is no visible agency applying the force then the charge must be moving in a $\vec{B}$-field.

Problem I: At t=0, charge q is at the origin and has velocity $\vec{v} = v\hat{x}$. Turn on a field $\vec{B} = B\hat{z}$ immediately, it experiences $\vec{F}_B$ along-$\hat{y}$. This makes $\vec{v}$ turn, but $\vec{F}_B$ turns also. Net result is as shown in Figure. q goes around in a circle $\vec{F}_B \perp \vec{v}$ always so Kinetic Energy is fixed, magnitude of $\vec{v}$ does not change. Since $\vec{F}_B \perp \vec{v}$, $\vec{F}_B$ cannot do work on q. Direction of $\vec{v}$ changed magnitude of $\vec{v}$ does not!





Particle moves under influence of $\vec{F}_B = -qvB\hat{r}$ [$\vec{v}$ & $\vec{B}$ *are* $\perp$ *to one another*]

Note: Plane of orbit $\perp$ to $\vec{B}$ field.

Note: Uniform circular motion needs a centripetal force.

$$\vec{F}_C = \frac{-Mv^2}{r}\hat{r}$$

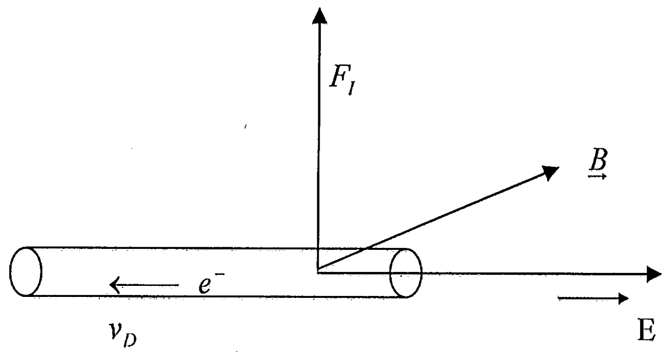
$\vec{F}_B$ provides it

$$\vec{F}_B = \vec{F}_C \quad so \quad r = \frac{Mv}{qB}$$

angular velocity $\vec{\omega} = \frac{-qB}{m}\hat{z}$ (see picture above)

Note: ω independent of v.

Problem II: Force on Current Carrying conductor of length l; Cross. Sec A, charge density n_e each electron feels $F_B = (-e)[\underset{\rightarrow}{v_D} \times \underset{\rightarrow}{B}]$



of $electrons = n_e A \Delta l$

so total force on conductor $\underset{\rightarrow}{F_I} = n_e(-e)A\Delta l[\underset{\rightarrow}{v_D} \times \underset{\rightarrow}{B}] = I \underset{\rightarrow}{\Delta l} \times \underset{\rightarrow}{B}$

electrons constrained to move along Δl so $\underset{\rightarrow}{F_I} = n_e e A v_D [\underset{\rightarrow}{\Delta l} \times \underset{\rightarrow}{B}] = I \underset{\rightarrow}{\Delta l} \times \underset{\rightarrow}{B}$

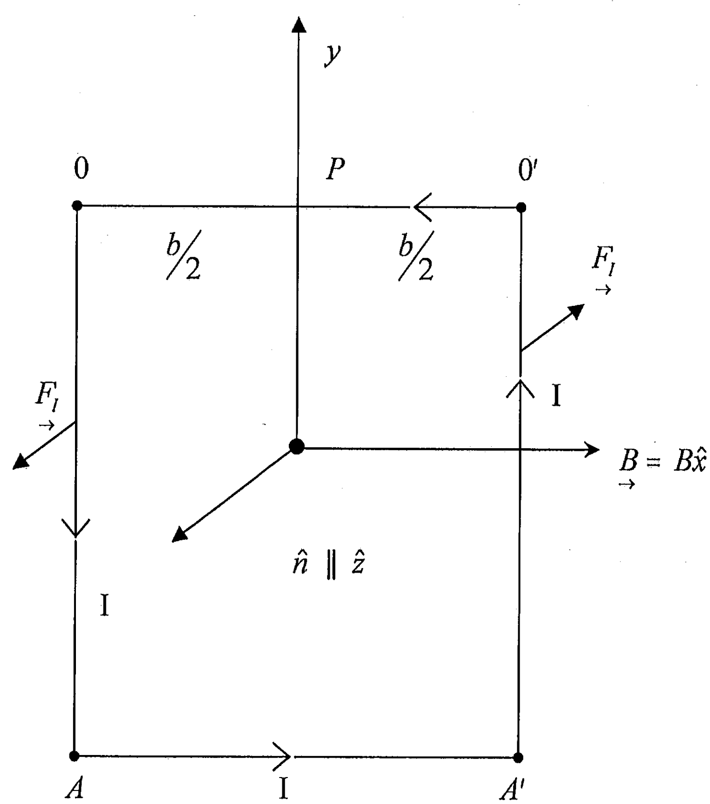
Problem III

Rectangular loop of wire suspended in a $\underset{\rightarrow}{B}$-field with current in loop as shown, start with

loop in
xy-pl, at t=0.

$\underset{\rightarrow}{F_I} = I l B \hat{z}$ on $0A$

$\underset{\rightarrow}{F_I} = -I l B \hat{z}$ on $0'A'$

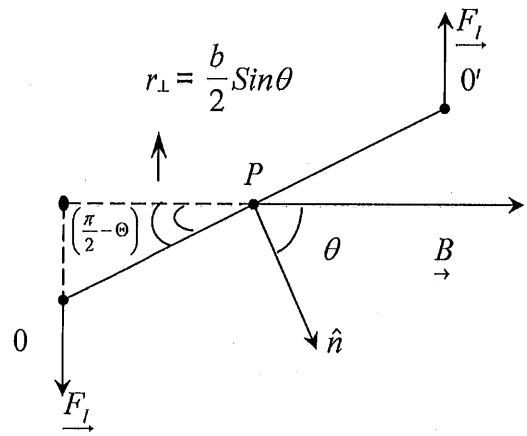


Net force is zero. However, torque is given by

$$\underset{\rightarrow}{\tau} = I\,l\,B\frac{b}{2}\hat{y} + I\,l\,B\frac{b}{2}\hat{y} \qquad (1)$$

$$= I\,l\,B\,b\hat{y}$$

<u>Rotate</u> (look from above)



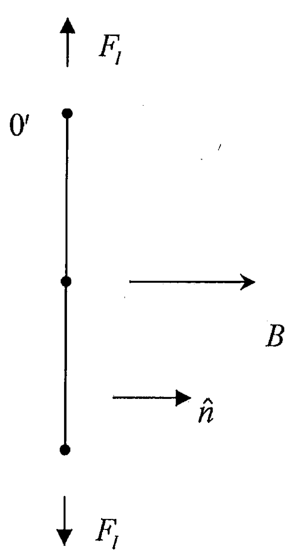
$\underset{\rightarrow}{\tau} = [\underset{\rightarrow}{r} \times \underset{\rightarrow}{F}]$

$\tau = r_\perp F$

Note I and B still at right angles to one another, F_l does not change but now $r_\perp = \frac{b}{2}Sin\theta$.

[Direction of $\hat{n}$ also fixed by right hand rule] $\underset{\rightarrow}{\tau} = IlBb\,Sin\theta\,\hat{y}$ (2)

Rotate further:



$\underset{\rightarrow}{\tau} = 0$ (3)

$\hat{x} \times \hat{y} = \hat{z}$

$\hat{y} \times \hat{z} = \hat{x}$

$\hat{z} \times \hat{x} = \hat{y}$

Equations (1), (2), (3) combine to give $\underset{\rightarrow}{\tau} = Ilb\hat{n} \times \underset{\rightarrow}{B}$

Define Magnetic (Dipole) moment $\vec{\mu} = Ilb\hat{n} = IA\hat{n}$

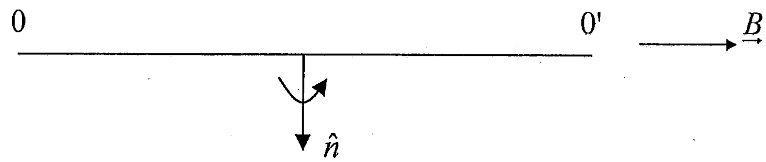
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

If the coil has N turns $\vec{\mu} = IAN\hat{n}!$

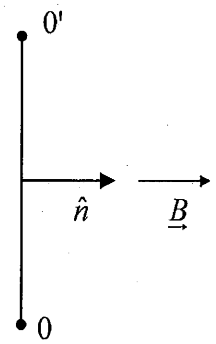
Note: The top $(00')$ and bottom (AA') wires have equal and opposite Forces. They will make the coil out of shape but have no other effect.

The above discussion forms the basis of a dc Motor, It is best to look at it as follows. First, we establish I such that we have the situation.

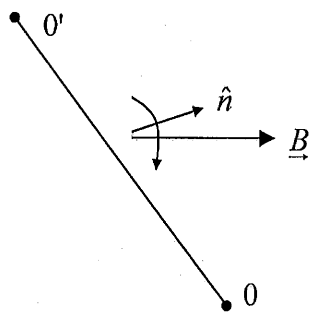
$\vec{\mu} = IA\hat{z}$, $\vec{\tau} = \vec{\mu} x \vec{B}$



And the torque will cause the coil to turn counterclockwise until

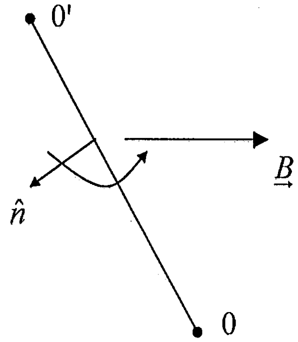


If we do not switch I the coil overshoots



But now torque will bring it back toward $\vec{B}$ (clockwise)

Therefore, in order to keep it going we reverse I and hence $\hat{n}$ to give



And the torque will turn $\hat{n}$ toward $\underset{\to}{B}$, counter clockwise so for a dc motor we established a current in a coil, suspended in a $\underset{\to}{B}$ field, and free to rotate about an axis perpendicular to $\underset{\to}{B}$, and reverse the current every half cycle using a commutator and brushes.

Shown schematically along side. In actual motors, there are many current loops and commutators for smooth running.